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# Improved sqrt-cosine similarity measurement

Sahar Sohangir\* o and Dingding Wang

\*Correspondence: ssohangir2014@fau.edu Department of Computer & Electrical Engineering and Computer Science, Florida Atlantic University, Boca Raton, FL 33431, USA

#### **Abstract**

Text similarity measurement aims to find the commonality existing among text documents, which is fundamental to most information extraction, information retrieval, and text mining problems. Cosine similarity based on Euclidean distance is currently one of the most widely used similarity measurements. However, Euclidean distance is generally not an effective metric for dealing with probabilities, which are often used in text analytics. In this paper, we propose a new similarity measure based on sqrtcosine similarity. We apply the proposed improved sqrt-cosine similarity to a variety of document-understanding tasks, such as text classification, clustering, and query search. Comprehensive experiments are then conducted to evaluate our new similarity measurement in comparison to existing methods. These experimental results show that our proposed method is indeed effective.

Keywords: Similarity measure, Information retrieval, Hellinger distance

#### Introduction

In the past decade, there has been explosive growth in the volume of text documents flowing over the internet. This has brought about a need for efficient and effective methods of automated document understanding, which aims to deliver desired information to users. Document similarity is a practical and widely used approach to address the issues encountered when machines process natural language. Some examples of document similarity are document clustering, document categorization, document summarization, and query-based search.

Similarity measurement usually uses a *bag of words* model [1]. This model views a document as a collection of words and disregards grammar and word order. For example, consider that we want to compute a similarity score between two documents, t and d. One common method for similarity measurement is to first assign a weight to each term in the document by using the number of times the term occurs, then invert the number of occurrences of the term in all documents ( $tfidf_{t,d}$ ) [2, 3], and finally calculate the similarity based on the weighting results using a *vector space* model [4]. In a *vector space scoring* model, each document is viewed as a *vector* and each term in the document corresponds to a component in vector space. Another popular and commonly-used similarity measure is *cosine similarity*. This can be derived directly from Euclidean distance,



however, Euclidean distance is generally not a desirable metric for high-dimensional data mining applications.

In this paper, we propose a new similarity measurement based on Hellinger distance. Hellinger distance (L1 norm) is considerably more desirable than Euclidean distance (L2 norm) as a metric for high-dimensional data mining applications [5]. We conduct comprehensive experiments to compare our newly proposed similarity measurement with the most widely used cosine and Gaussian model-based similarity measurements in various document understanding tasks, including document classification, document clustering, and query search.

#### **Related work**

Block distance, which is also known as Manhattan distance, computes the distance that would be traveled to get from one data point to the other if a grid-like path is followed. The Block distance between two items is the sum of the differences of their corresponding components [6]. Euclidean distance, or L2 distance, is the square root of the sum of squared differences between corresponding elements of the two vectors. Matching coefficient is a very simple vector based approach which simply counts the number of similar terms (dimensions), with which both vectors are non-zero. Overlap coefficient considers two strings as a full match if one is a subset of the other [7]. Gaussian model is a probabilistic model which can be used to characterize a group of feature vectors of any number of dimensions with two values, a mean vector, and a covariance matrix. The Gaussian model is one way of calculating the conditional probability [8]. Traditional spectral clustering algorithms typically use a Gaussian kernel function as a similarity measure. Kullback-Leibler divergences [9] is another measure for computing the similarity between two vectors. It is a non-symmetric measure of the difference between the probability distribution correspond with the two vectors [10]. The Canberra distance metric [11] is always used in a non-negative vector. Chebyshev distance is defined on a vector space where the distance between two vectors is the greatest of difference along any coordinate dimension [12]. Triangle distance is considered as the cosine of a triangle between two vectors and its value range between 0 and 2 [13]. The Bray-Curtis similarity measure [14] which is sensitive to outlying values is a city-block metric. The Hamming distance [15, 16] is the number of positions at which the associated symbols are different. IT-Sim (information-theoretic measure) for document similarity, proposed in [17, 18]. The Suffix Tree Document (STD) model [19] is a phrase-based measure.

Also, there are some similarity measures which incorporate the inner product in their definition. The inner product of two vectors yields a scalar which is sometimes called the dot product or scalar product [20]. In [21] Kumar and Hassebrook used inner product to measures the Peak-to-correlation energy (PCE). Jaccard coefficient also called Tanimoto [22] is also the normalized inner product. Jaccard similarity is computed as the number of shared terms over the number of all unique terms in both strings [23]. Dice coefficient [24] also called Sorensen, Czekannowski Hodgkin-Richards [25] or Morisita [26]. Dice's coefficient is defined as twice the number of common terms in the compared strings divided by the total number of terms in both strings [27]. Cosine coefficient measures the angle between two vectors. It is the normalized inner product and also called Ochiai [28] and Carbo [25]. Some similarity measure like soft cosine measure

proposed in [29] takes into account similarity of features. They add to the Vector Space Model new features by calculation of similarity of each pair of the already existing features. Pairwise-adaptive similarity dynamically select number of features prior to every similarity measurement. Based on this method a relevant subset of terms is selected that will contribute to the measured distance between both related vectors [30].

#### Cosine similarity

Similarity measurement is a major computational burden in document understanding tasks and cosine similarity is one of the most popular text similarity measures. Manning and Raghavan provide an example in [2] which clearly demonstrates the functionality of cosine similarity. In this example, four terms (affection, jealous, gossip, and wuthering) from the novels *Sense and Sensibility (SaS)* and *Pride and Prejudice (PaP)* from Jane Austen and *Wuthering Heights (WH)* from Emily Bronte are extracted. For the sake of simplicity we ignore *idf* and use Eq. 1 to calculate log frequency weight of term t in novel d.

$$w_{t,d} = \begin{cases} 1 + \log_{10} t f_{t,d} & \text{if } t f_{t,d} > 0\\ 0 & \text{otherwise} \end{cases}$$
 (1)

In Tables 1 and 2, the number of occurrences and log frequency weight of these terms in each of the novels are provided, respectively. Table 3 then shows the cosine similarity between these novels. Cosine similarity returns one when two documents are practically identical, or zero when the documents are completely dissimilar.

Table 1 Term frequencies of terms in each of the novels

Term	SaS	PaP	WH
Affection	115	58	20
Jealous	10	7	11
Gossip	2	0	6
Wuthering	0	0	38

Table 2 Log frequency weight of terms in each of the novels

Term	SaS	PaP	WH
Affection	3.06	2.76	2.30
Jealous	2.00	1.85	2.04
Gossip	1.30	0	1.78
Wuthering	0	0	2.58

Table 3 Cosine similarity between novels

	SaS	PaP	WH
SaS	1.00	0.94	0.79
PaP	0.94	1.00	0.69
WH	0.79	0.69	1.00

In order to find cosine similarity between two documents x and y we need to normalize them to one in  $L_2$  norm (2).

$$\sum_{i=1}^{m} x_i^2 = 1 \tag{2}$$

By having two normalized vectors x and y the cosine similarity between them will be simply the dot product of them (Eq. 3).

$$\cos(x,y) = \frac{\sum_{i=1}^{m} x_i y_i}{\sqrt{\sum_{i=1}^{m} x_i^2} \sqrt{\sum_{i=1}^{m} y_i^2}}.$$
(3)

Careful examination of Eq. 3 shows that cosine similarity is directly derived from Euclidean distance (Eq. 4).

$$d_{\text{Euclid}}(x,y) = \left[\sum_{i} (x_i - y_i)^2\right]^{\frac{1}{2}} = \left[2 - 2\sum_{i} x_i y_i\right]^{\frac{1}{2}}$$
(4)

#### Sqrt-cosine similarity

Zhu et al. in [31] attempted to use the advantages of Hellinger distance Eq. (6) and proposed a new similarity measurement—sqrt-cosine similarity. They claim that as a similarity measurement, it provides a value between zero and one, which is better assessed with probability-based approaches. However, Euclidean distance is not a good metric to deal with probability. The sqrt-cosine similarity defined in Eq. (5) is based on Hellinger distance Eq. (6).

$$\operatorname{SqrtCos}(x, y) = \frac{\sum_{i=1}^{m} \sqrt{x_i y_i}}{\left(\sum_{i=1}^{m} x_i\right) \left(\sum_{i=1}^{m} y_i\right)}.$$
 (5)

$$H(x,y) = \left[\sum_{i}^{m} \left(\sqrt{x_i} - \sqrt{y_i}\right)^2\right]^{\frac{1}{2}} = \left[2 - 2\sum_{i}^{m} \sqrt{x_i y_i}\right]^{\frac{1}{2}},\tag{6}$$

In some cases, the manner of sqrt-cosine similarity is in conflict with the definition of similarity measurement. To clarify our claim, we use the same example provided in "Cosine similarity". Sqrt-cosine similarity is calculated between these three novels and shown in Table 4. Surprisingly, the sqrt-cosine similarity between two equal novels does

Table 4 Sqrt-cosine similarity scores among novels

	SaS	PaP	WH
SaS	0.15	0.16	0.11
PaP	0.16	0.21	0.11
WH	0.11	0.11	0.11

not equal one, exposing flaws in this design. Furthermore, from on Table 4, we can see that the SaS (Sense and Sensibility) novel is more similar to PaP (Pride and Prejudice) than itself! comparing Tables 3 and 4 reveals that, opposed to cosine similarity, we cannot specify the sqrt-cos similarity of WH (Wuthering Heights) to other novels within two decimal places of accuracy. Based on the above example we believe that sqrt-cosine similarity is not a trustable similarity measurement. To address this problem, we propose an improved similarity measurement based on sqrt-cosine similarity and compare it with other common similarity measurements.

### The proposed ISC similarity

Information retrieved from high-dimensional data is very common, but this space becomes a problem when working with Euclidean distances. In higher dimensions, this can rarely be considered an effective distance measurement in machine learning.

Charu, in [5], prove that the Euclidean (L2) norm, from a theoretical and empirical view, is often not a desirable metric for high-dimensional data mining applications. For a wide variety of distance functions, because of the concentration of distance in high-dimensional spaces, the ratio of the distances of the nearest and farthest neighbors to a given target is almost one. As a result, there is no variation between the distances of different data points. Also in [5], Charu investigates the behavior of the  $L_k$  norm in high-dimensional space. Based on these results, for a given value of the high-dimensionality d, it may be preferable to use a lower value of k. In other words, for a high-dimensional application,  $L_1$  distance, like Hellinger, is more favorable than  $L_2$  (Euclidean distance).

We propose our improved sqrt-cosine (ISC) similarity measurement below.

$$ISC(x,y) = \frac{\sum_{i=1}^{m} \sqrt{x_i y_i}}{\sqrt{\left(\sum_{i=1}^{m} x_i\right)} \sqrt{\left(\sum_{i=1}^{m} y_i\right)}}.$$
(7)

In Eq. 5, each document is normalized to 1 in  $L_1$  norm:  $\sum_{i=1}^{m} x_i = 1$ . We propose the ISC similarity measurement in Eq. 7. In this equation, instead of using  $L_1$  norm, we use the square root of  $L_1$  norm.

The same example is used in "Cosine similarity" to compare our ISC similarity with the previous one. The results from using ISC similarity between these three novels are available in Table 5. The similarity between two identical novels is one and we can clearly find a similar novel to WH (Wuthering Heights) within two decimal place of accuracy.

Cosine similarity is considered as the "state of the art" in similarity measurement. ISC is very close to cosine similarity in term of implementation complexity in major engines such as Spark [32] or any improved big data architecture [33, 34]. We conduct comprehensive experiments to compare ISC similarity with cosine similarity and Gaussian model-based similarity in various application domains, including document classification, document clustering, and information retrieval query. In this study, we used several popular learning algorithms and applied them to multiple data sets. We also use various evaluation metrics in order to validate and compare our results.

Table 5 The proposed ISC similarity scores among novels

	SaS	PaP	WH
SaS	1.00	0.89	0.83
PaP	0.89	1.00	0.70
WH	0.83	0.70	1.00

# **Experiments**

#### Data sets

Five different data sets from different application domains were used in this experiment. In Table 6, a list of these sets is presented. Our reason for selecting these data sets is that they are commonly used and considered a benchmark for document classification and clustering. In the following table, more information about all the data sets used in our experiments can be found.

- The CSTR is a collection of about 550 abstracts of technical reports published from 1991 to 2007 in computer science journals at the University of Rochester. They can be classified into four different groups: Natural Language Processing (NLP), Robotics/vision, Systems, and Theory.
- 2. The *DBLP* data set contains the titles of the last 20 years' papers, published by 552 active researchers, from nine different research areas: database, data mining, software engineering, computer theory, computer vision, operating systems, machine learning, networking, and natural language processing.
- 3. *Reuters*-21578 is a collection of documents that appeared on the Reuters newswire in 1987. *R8* and *R52* are subsets of the Reuters-21578 Text Categorization. Reuters Ltd personnel collected this document set and labeled the contents.
- 4. The *WebKB* data set contains 8280 documents which are web pages from various college computer science departments. These documents are divided into seven groups: student, faculty, staff, course, project, department, and other. The four most popular categories from these seven categories are selected and made into the WebKB4 set. These four categories are student, faculty, course and project [36].
- 5. The *20 Newsgroups* data set is a collection of about 20 different newsgroups [37]. Containing around 20,000 newsgroup documents, this is one of the most commonly-used data sets in text processing.

#### Learners

We apply various classification and clustering methods to analyze the performance of our new similarity measurement. We used Nearest Neighbour [38], Naïve Bayes [39]

Table 6 Summary of the real-world data sets [35]

	#Sample	#Dim	#Class
CSTR	475	1000	4
DBLP	1367	200	9
Reuters	2900	1000	8-52
WebKB4	4199	1000	4
Newsgroups	11293	1000	20

and Support Vector Machine [40] which are most common classification models. As the clustering models we used K-Means [41], Normalized Cut Algorithm [42], K-means Clustering via Principal Component Analysis [43]. We implemented discussing learners in R language [44].

#### **Performance metrics**

In the experiments, we use five different performance metrics to compare the models we constructed based on our ISC similarity with other similarity measures. The evaluation metrics include the following.

Area under the ROC Curve [45], accuracy for classification [46], accuracy for clustering [47], purity [2], and normalized mutual information [48].

In addition to these performance metrics, we test the results for statistical significance at the  $\alpha=5\%$  level using a one-factor analysis of variance (ANOVA) [49]. An ANOVA model can be used to test the hypothesis that classification performances for each level of the main factor(s) are equal versus the alternative hypothesis that at least one is different. In this paper, we use a one-factor ANOVA model, which can be represented as:

$$\psi_{jn} = \mu + \theta_j + \epsilon_{jn} \tag{8}$$

where  $\psi_{jn}$  represents the response (i.e., AUC, ACC, Purity, NMI) for the nth observation of the jth level of experimental factor  $\theta$ ;  $\mu$  represents overall mean performance;  $\theta_j$  is the mean performance of level j for factor  $\theta$ ; and  $\epsilon_{jn}$  is random error.

In our experiment,  $\theta$  is the similarity measure and we aim to compare the average performance of the newly proposed similarity measurement with cosine similarity and Gaussian-based similarity measurement. If at least one level of  $\theta$  is different, there are lots of procedures exist that can be used to specify which levels of  $\theta$  is different. In this paper, we use Tukey's Honestly Significant Difference (HSD) test [50] to identify which levels of  $\theta$  are significantly different.

#### **Experimental results**

In this section, we provide the results of our experiments and compare our ISC similarity with cosine similarity and Gaussian base similarity. As a first step, we just focus on the performance metrics across all five data sets and seven different learners (three classifications and four clustering models). As a second step, we consider different learners separately to compare the performance of these similarity measurements for different learners. At the end, the combinations of learners and data sets considered seeing their effectiveness.

#### Overall results

First, we compare the average performance of our proposed ISC similarity measurement with cosine similarity and Gaussian-based similarity measurement. Results are provided in Tables 7 and 8. Mean columns represent the average of performance metrics across all learners (clustering and classification) and data sets. According to the mean values in Tables 7 and 8, ISC similarity in all cases outperforms cosine similarity and Gaussian-based similarity measurement.

Table 7 Average performances of the similarity measure across all clustering learners and data sets

Similarity	Accuracy		Purity		NMI		
	Mean	HSD	Mean	HSD	Mean	HSD	
ISC	0.3563	A	0.5950	A	0.1590	А	
Cosine	0.3370	Α	0.5608	Α	0.1363	Α	
Gaussian	0.2949	Α	0.5597	Α	0.0990	Α	

Table 8 Average performance of the similarity measure across all classification learners and data sets

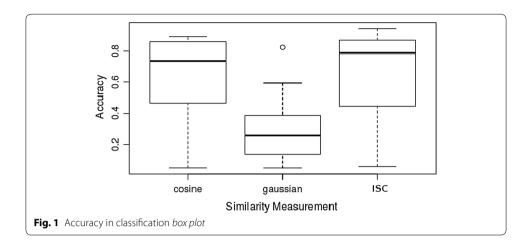
Similarity	Accuracy		AUC	
	Mean	HSD	Mean	HSD
ISC	0.6562	A	0.7901	А
Cosine	0.6371	А	0.7780	Α
Gaussian	0.2872	В	0.5582	В

Columns labeled HSD represent results of Tukey's Honestly Significant Difference test at the 95% confidence level. If two similarity measurements have the same letter in the HSD column, then according to HSD test their average performances are both good. For example, based on Table 7, using area under the ROC curve (AUC) or accuracy as a performance measure of each classifier indicates that ISC and cosine similarity are in the same group so their performance are not significantly different from each other. On Table 8 Gaussian-based similarity belongs to group B which means ISC and cosine similarity outperform Gaussian-based similarity. Generally speaking, based on the HSD test, when averaging performance across all data sets and learners, the proposed ISC similarity, and cosine similarity belong to the same group.

In addition, we use box plots to see outliers and spread the performance of classification and clustering across all data sets and learners for these three similarity measurements. In this way, we can compare their performance at various points in the distribution, not only the mean value as we did in Tables 7 and 8. For example, based on Figs. 1 and 2, the distribution of the accuracy and purity for ISC similarity is more favorable than those of cosine similarity and Gaussian.

# Results using different learners

As a second step, we try to compare the effectiveness of different learners on the performance of our ISC similarity, cosine similarity, and Gaussian-based similarity measurement. Tables 9 and 10 show the average performance of discussing similarity measurements by applying different classification and clustering methods. We used K-Nearest Neighbor, Naïve Bayes, and SVM as our classification models. K-Means, Normalized cut, K-Mean clustering via Principal Component Analysis and Symmetric Nonnegative Matrix Factorization (SymNMF) are our applying clustering methods. We summarized our observations as below:



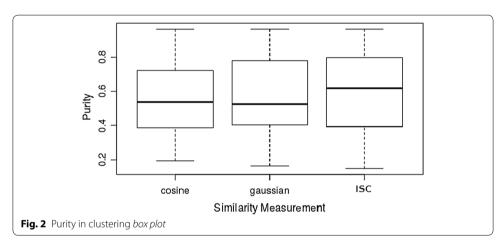


Table 9 Performances of the similarity measures using classification learners averaged across all data sets

Metric	Similarity	KNN	KNN		Naïve Bays		SVM	
		Mean	HSD	Mean	HSD	Mean	HSD	
Accuracy	ISC	0.7079	А	0.8589	А	0.4019	А	
Cosine	Cosine	0.6476	Α	0.8633	Α	0.4004	Α	
	Gaussian	0.4606	Α	0.1795	В	0.2215	Α	
AUC	ISC	0.8779	Α	0.8806	Α	0.612	Α	
Cosine Gaussian	Cosine	0.7977	AB	0.8892	Α	0.6473	Α	
	Gaussian	0.6620	В	0.5084	В	0.5042	Α	

- 1. With naïve Bays as the base learner and using accuracy and area under the ROC Curve (AUC) to measure performance, ISC similarity and cosine similarity are preferred over Gaussian base similarity measurement.
- 2. Based on mean values, ISC similarity is preferred over cosine similarity and Gaussian-based similarity measurement.
- 3. Based on the HSD test, both ISC similarity and cosine similarity are belong to group 'A' which is the top grade ranges.

Table 10 Performances of the similarity measures using clustering learners averaged across all data sets

Metric	Similarity	Kmeans		Ncut	Ncut		ean	SYM-NMF	
		Mean	HSD	Mean	HSD	Mean	HSD	Mean	HSD
Accuracy	ISC	0.3354	А	0.3220	А	0.3090	А	0.4589	Α
	Cosine	0.3115	Α	0.3104	Α	0.3070	Α	0.4191	Α
	Gaussian	0.3005	Α	0.3020	Α	0.3020	Α	0.2750	Α
Purity	ISC	0.4357	Α	0.5606	Α	0.8499	Α	0.5337	Α
	Cosine	0.4217	Α	0.5626	Α	0.7771	Α	0.5072	Α
	Gaussian	0.3919	Α	0.5693	Α	0.8457	Α	0.4066	Α
NMI	ISC	0.1740	Α	0.1367	Α	0.0369	Α	0.2886	Α
	Cosine	0.1332	Α	0.1321	Α	0.0335	Α	0.2464	Α
	Gaussian	0.0992	Α	0.1337	Α	0.0309	Α	0.1321	Α

#### Results using different data sets and learners

In this section, we try to investigate the effectiveness of different data sets in various domains on the performance of discussing similarity measurement. We consider six different data sets from different application domains including Webkb, Reuters8, Reuters52, News, dblp and cstr data sets. Table 11 shows the results of evaluating all classification methods and Table 12 presents the results of clustering methods. We use various performance evaluations for both classification and clustering. For each performance metric, we specify a row which represents the number of data sets where the given technique is in group A and also the average performance across all six data sets. Based on Tables 11 and 12 regardless of learners and data sets, ISC similarity and cosine similarity measures are always in group A. On the other hand, the Gaussian-based similarity measurement is in group B for some data sets while we use classification learners.

Table 11 Performance of the similarity measures in data sets averaged across all classification learners

Metric	Data set	ISC		Cosine		Gaussian	
		Mean	HSD	Mean	HSD	Mean	HSD
Accuracy	WEBKB	0.6104	А	0.5929	А	0.3046	А
	R8	0.7166	Α	0.7361	Α	0.4485	Α
	R52	0.4975	Α	0.4230	Α	0.1945	Α
	NEWS	0.6009	Α	0.5989	Α	0.2468	Α
	DBLP	0.7101	Α	0.6842	Α	0.2234	В
	CSTR	0.8019	Α	0.7873	Α	0.3052	В
Average #A's		0.6562	6	0.6370	6	0.2871	4
AUC	WEBKB	0.8162	Α	0.8304	Α	0.6171	Α
	R8	0.7342	Α	0.6641	Α	0.5341	Α
	R52	0.7826	Α	0.7540	Α	0.5075	Α
	NEWS	0.7514	Α	0.7570	Α	0.5852	Α
	DBLP	0.9253	Α	0.9287	Α	0.6011	В
	CSTR	0.7313	Α	0.7340	Α	0.5040	Α
Average #A's		0.7901	6	0.7780	6	0.5581	5

Table 12 Performance of the similarity measures in data sets averaged across all clustering learners

Metric	Data set	ISC		Cosine		Gaussian	
		Mean	HSD	Mean	HSD	Mean	HSD
Accuracy	WEBKB	0.4798	А	0.4434	А	0.3824	А
	R8	0.4384	Α	0.4291	Α	0.4472	Α
	R52	0.2320	Α	0.2283	Α	0.2395	Α
	NEWS	0.1659	Α	0.1544	Α	0.1179	Α
	DBLP	0.3886	Α	0.3574	Α	0.2640	Α
	CSTR	0.4332	Α	0.4095	Α	0.3182	Α
Average #A's		0.3563	6	0.3370	6	0.2948	6
Purity	WEBKB	0.6248	Α	0.6091	Α	0.5548	Α
	R8	0.5769	Α	0.5790	Α	0.6446	Α
	R52	0.4440	Α	0.4225	Α	0.4478	Α
	NEWS	0.4234	Α	0.4410	Α	0.3948	Α
	DBLP	0.7980	Α	0.6363	Α	0.6531	Α
	CSTR	0.7026	Α	0.6704	Α	0.6700	Α
Average #A's		0.59495	6	0.55971	6	0.56085	6
NMI	WEBKB	0.1500	Α	0.1177	Α	0.0879	Α
	R8	0.1978	Α	0.1912	Α	0.2030	Α
	R52	0.1376	Α	0.1321	Α	0.1179	Α
	NEWS	0.0855	Α	0.0761	Α	0.0731	Α
	DBLP	0.2439	Α	0.1940	Α	0.0948	Α
	CSTR	0.1396	Α	0.1069	Α	0.0172	Α
Average #A's		0.1590	6	0.1363	6	0.0990	6

According to the average performance across all data sets in these Tables, regardless of learners, data sets or even quality measurement, ISC similarity always outperforms Gaussian-based and also cosine similarity measure.

#### **Conclusion**

Finding an effective and efficient way to calculate text similarity is a critical problem in text mining and information retrieval. One of the most popular similarity measures is cosine similarity, which is based on Euclidean distance. It has been shown useful in many applications, however, cosine similarity is not ideal. Euclidean distance is based on L2 norm and does not work well with high-dimensional data. In this paper, we proposed a new similarity measurement technique, called improved sqrt-cosine (ISC) similarity, which is based on Hellinger distance. Hellinger distance is based on L1 norm and it is proven that in high-dimensional data, L1 norm works better than L2 norm. Most applications consider cosine similarity "state of the art" in similarity measurement. We compare the performance of ISC with cosine similarity, and other popular techniques for measuring text similarities, in various document understanding tasks. Through comprehensive experiments, we observe that although ISC is very close to cosine similarity in term of implementation, it performs favorably when compared to other similarity measures in high-dimensional data.

#### Authors' contributions

DW brought up the idea of new similarity based on Hellingerdistance when work on the problem of cosine similarity in high-dimensional data. She found that Hellinger distance is a good choice to replace Euclidean distance in cosine

similarity. Sahar Sohangir prposed improved sqrt-cosine similarity. She did a comprehensive study to compare this new similarity measurement with other popular similarity measurements. Both authors read and approved the final manuscript.

#### Acknowledgements

Not applicable.

#### **Competing interests**

The authors declare that they have no competing interests.

# **Publisher's Note**

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 2 May 2017 Accepted: 29 June 2017 Published online: 25 July 2017

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