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Metamorphosing forex: advancements in volatility forecasting using a modified fuzzy time series framework

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Abstract

The interplay of exchange rates among nations significantly influences both international and domestic trade, underscoring the pivotal role of the foreign exchange market (Forex) in a country's economic landscape. Forex fluctuations have a significant impact on the everyday lives of both government agencies and the public population, directly influencing a country's prosperity or misfortune. This work proposes an advanced fuzzy time series model that incorporates domain universe sub-partitioning, parameter adjustment optimization methodologies, and interval forecasting methods. We utilized this model to examine annual exchange rate patterns between the Pakistani rupee (PKR) and the US dollar (US$), comparing its forecast accuracy to that of other models. Our proposed methodology outperformed existing methodologies in terms of forecasting precision, providing stakeholders with valuable insights for making informed, data-driven business decisions that benefit both individual firms and the country overall.

Keywords: Forex, Fuzzy time series, Autoregressive integrated moving average, Average forecast error rate

Introduction

The exchange rate (ER) is an important economic statistic that affects many elements of a country's economy, commerce, and business. Stakeholders closely monitor current ER trends and predictions, recognizing the value of informed decision-making. As a result, creating a stronger currency exchange rate forecasting methodology is crucial for businesses looking to decrease uncertainty. The foreign currency market has a considerable impact on the global economy due to its interconnectedness. The authors [1], acknowledging Pakistan as a developing country in South Asia with economic challenges, note frequent fluctuations in the exchange rate between the Pakistani rupee (PKR) and the US dollar (USD). The currency rate's stability is crucial for long-term economic prosperity. A rise in the PKR-USD exchange rate boosts the value of outstanding debt while also disrupting the balance of payments equilibrium. This hurts the overall population leading to inflationary pressures on everyday consumer goods. It stresses the need for policymakers to regularly monitor the exchange rate to minimize negative deviations since these changes can have a cascading impact on several financial indices [2].

Currency depreciation is a geopolitical tactic used by governments to deliberately reduce a country's currency value versus gold or other foreign currencies. This deliberate depreciation is frequently used to correct external sector imbalances and to promote economic development and stability. The justification for this strategy stems from its ability to increase the competitiveness of domestic goods and services in foreign markets. A depreciated currency can effectively reduce the cost of exports, making them more appealing to overseas purchasers and increasing export profits. This can result in better output, job creation, and overall economic growth. Currency depreciation is often regarded as a feasible approach for economic revitalization. International financial organizations, such as the IMF, acknowledge its potential for increasing a country's external competitiveness. Furthermore, as the custodian of monetary policy, the Central Bank has continuously emphasized the significance of implementing policies that promote export growth and reduce import reliance. In this line, the IMF and Central Bank recommend a multimodal strategy to improve external competitiveness. This includes not just endeavors to increase export quantities, but also joint efforts to reduce import costs. Nations can enhance their current account balances and hence their ability to service foreign loans by attaining a more favorable trade balance.

The call for enhancing external competitiveness resonates strongly in the context of today's global economy, characterized by heightened interconnectivity and competition. In the face of mounting economic challenges, nations must proactively implement policies aimed at fortifying their export sectors and reducing reliance on imports. By doing so, they can navigate the complexities of the international economic landscape with greater resilience and ensure sustained prosperity for their citizens. PKR vs USD has recently fallen to a historic low of Rs. 176.50, due to significant demand for import payments. However, the depreciation has resulted in higher import costs and localized inflation, reducing the general public's purchasing power. It is critical to note that, while local currency depreciation is a temporary solution to increase competitiveness, it is not a long-term plan for economic growth. Over time, it can undermine the overall health of the country's economy when compared to other nations. Therefore, adopting a more comprehensive approach to economic development is imperative for sustained and robust growth [2]. Pakistan is dealing with a variety of socioeconomic issues, emphasizing the need for monetary policymakers to use forecasting techniques to make educated, data-driven choices. Accurate forecasting allows politicians to change and revise policies based on changing conditions, giving a proactive and adaptable approach to managing the intricacies of the economy [3].

The PKR-US dollar exchange rate has generally increased over time. Before 2008, the US dollar traded in the interbank market for Rs. 45. However, the exchange rate has risen by more than 30% since 2000. Subsequent governments also saw considerable fluctuations: under Asif Zardari's term, it increased to Rs. 95, while it peaked at more than Rs. 120 during Nawaz Sharif's tenure. Presently, under Imran Khan's government, the current exchange rate stands at approximately Rs. 179.18, with predictions indicating a potential further increase. During the early days of the COVID-19 epidemic, there was a minor declining tendency in the exchange rate, but it was insignificant. Forecasting, considered a vital technique in research, involves making claims about the future. Researchers in this study have utilized the PKR-US $ exchange rate as a variable for predictive purposes. The key goal of this research is to create a complete forecasting model for analyzing the PKR-US$ exchange rate. It's worth noting that many forecasting models place limits on variables, such as the Markovian property, monotonous trends, scalarity, stationarity, and other features, which influence the dynamics of economy indices.

The key contributions of this study include:

1. Significant changes in UoD.
2. Proposed fuzzy time series model.
3. Point and Interval estimation for forecasting.
4. Relative efficiency.

Literature review

Zadeh's [4] groundbreaking work on fuzzy set theory has not only significantly influenced various research domains but has also made a notable impact on handling uncertainty in time series data. Fuzzy approaches have proven to enhance the predictability of real-world processes. In the contemporary scientific landscape, forecasting plays a pivotal role across diverse business sectors, managed by both individuals and companies alike. Data scientists contribute to well-informed, data-driven decision-making by minimizing forecast error rates, thereby assisting businesses in navigating the complexities of an ever-changing landscape. The inherent uncertainty in data, as well as the changing character of the nation's economic regulations, whether domestic or foreign, are strong considerations motivating businesspeople to embrace forecasting approaches. This strategic approach aims to mitigate the risk of substantial losses in their ventures. It is acknowledged that forecast results are not infallible, offering a future projection with a degree of uncertainty. While a 100% accurate prediction is unattainable, selecting an effective forecasting method becomes a critical consideration for businesses aiming to make prudent decisions in an unpredictable environment. Fuzzy set theory has proven to be a valuable tool in forecasting methods across a broad spectrum of scientific disciplines. The researchers [5] developed a simulation-based framework for predicting electric power demand-supply trends. This model included load components presented in language terms depending on different times of the day. These parameters were critical in assessing and selecting the best successful forecasting technology in the framework of electric power demand-supply patterns. The authors [6], investigation says that despite the protracted volatility in currency prices and multiple trade obstacles and found arbitrage pressures and resource re-allocation mechanisms are strong enough to ameliorate many of these distortions in the long run. They [7] examine the relationship between oil prices and inflation using RMB exchange rates and conclude that the covariance between exchange rates and oil prices is a reliable element in explaining and forecasting inflation in China.

The authors [8] introduced an advanced fuzzy forecasting model that is first-order and time-variant. They achieved this enhancement by utilizing frequency distribution partitioning of historical enrollment data. This approach allowed for a more nuanced and improved forecasting methodology, taking into account the temporal variations and distribution patterns present in the enrollment data. They [9] recommended a modified technique for assessing university enrollment statistics. They used the percentage change over consecutive years as the UoD in this way. The researchers [10] employed FTS and ARIMA models. The authors [11] performed a study of MYR-USD ER and found that the distance-based FTS model has superior reliability in forecasting ER. As indicated by [12], forecasting is a complex challenge characterized by substantial nonlinearity and irregularities in data. This complexity is particularly pronounced in predicting ERs, where reliance on inaccurate and inconsistent data adds another layer of difficulty. Analyzing forecasting models related to ER has consistently demonstrated sensitivity to market fluctuations. Consequently, forecasting ERs remains a challenging and intricate subject within the realm of finance. The authors [13] made predictions of current theoretical investigations on the effect of sanctions on the foreign exchange rate.

Using a performance indicator, they [14], studied the sufficiency of the features of the foreign exchange rate system in terms of its effectiveness. Notably, they explored how these features affected system effectiveness. Fuzzy inference methods have been frequently employed in the literature for time series forecasting, with an emphasis on their utility in capturing the inherent uncertainties and complexity associated with anticipating movements in foreign currency markets. [15]. The authors [16] conducted a comprehensive comparative study on the exchange rates between the PKR and US $. In addition to analyzing historical trends, they undertook the task of forecasting future exchange rates. To improve the precision of their predictions, they employed a combination of multiple forecasting models, aiming to enhance the overall accuracy and reliability of their projections for the Pak-US $ exchange rates [17]. He [2] specifically focused on forecasting the ER of PKR vs US$, employing the ARIMA model for this predictive analysis. The researcher [18] conducted a comparative analysis between the NTD and the US $ For linguistic analysis, he devised an FTS-Markov chain strategy, emphasizing the recommendation of using small sample time series data to enhance prediction accuracy further. The authors [19] introduced an innovative FTS forecasting approach integrating optimal UoD interval partitions and optimal Time Series Fuzzy Time-Lag Relevance Grades (TSFTLRG) weighting vectors. The investigators [20] delved into the daily exchange rate (ER) data, focusing on the US Dollar to Indonesian Rupiah (US $ to IDR), amidst its tumultuous volatility induced by the COVID-19 pandemic. They crafted a sophisticated forecasting model utilizing this ER dataset, with the objective of proactively preparing for and evaluating potential future scenarios shaped by the repercussions of the pandemic on exchange rates. They [21] compared the volatility forecasting capabilities of traditional time series models like ARIMA, EGARCH, and PARCH to fuzzy-based models. They employed these models against the backdrop of the Mexican Peso/US$ ER. The study [22], uses GARCH and EGARCH models to capture volatility and applies ANN to forecast British Pound and Indian Rupee exchange rates, with a hyperbolic tangent function in the hidden and output layer. The authors [23] believes that The Reserve Bank of India's sterilized interventions stabilize the market. An ARDL model shows a 1% rise in forex purchases depreciates the Rupee by 0.255% long-term, while short-term interventions reduce uncertainty. Better coordination with monetary policy is advised. The authors [24] created a model for forecasting Riyal against the Rupiah by using Indonesian Bank data. They carried out regulated forecasting for the next 10 days using the MCFTS technique and assessed accuracy using AFER and MEA criteria. Significant studies have been conducted on the importance of currency forecasting in determining international business risks and benefits [25]. Addressing the challenges of dynamic and complex data, the researchers introduced a novel weighted FTS model for daily USD to MYR ER forecasts. Comparisons with conventional methods demonstrated the proposed approach significantly improved forecast accuracy. The authors [26] attempted to redefine the UoD, calculating the base value to precisely determine the number of intervals. This approach was applied to simulate Indonesian annual petroleum production data, aiming to achieve targeted results. They forecasted the top price of shares for Bank Syariah Indonesia. [27]. They [28] proposed a novel approach called KmFuzz, utilizing modified K-Means clustering to determine an optimal number of partitions. This approach integrates fuzzy logic to predict power consumption for the U.P state, India, and is compared to various existing methods. The researchers [29–31] workout on forecasting methods for better accuracy while [32] have given a detailed study on past 30 years of ER analysis and forecasting.

The examples provided demonstrate the efficacy of FTS techniques in forecasting various research challenges and assisting decision-making. Building on the study [33], which utilized 17 years of annual data for Ukrainian Enterprise, our study employs 61 years of historical data for PKR to USD exchange rates to enhance the efficiency of our conclusions.

Research methodology

In this research study, we have proposed an FTS model in comparison to the FTS Stevenson and Porter Model referred to as the SP model [33], Autoregressive Integrated Moving Average (ARIMA), and other time series models. By using the percentage change of the dataset for forecasting as UoD, on which fuzzy sets are given, the interval of variations of such indicators as the year-to-year percentage change rates, called "chain growth rates".

Consider the following time series of an economic indicator, indicated by y_i, $i = 2, 3,\ldots n$. The proposed method utilizes the chain growth rate as (1):

$$T_i = \left(\frac{y_i}{y_i - 1} - 1\right) \times 100\,\% \tag{1}$$

Hereafter, the model encompasses the following procedure:

Step 1. Establishing the Universe of Discourse (UoD): UoD as the set U, where $U = [\min T_i, \max T_i]$, i = 2, 3,… n and utilize the frequency distribution class boundaries method to divide this set into "m" intervals (m = 7 in the referenced model).

Step 2. Division: Division of the larger classes/sets into smaller subdivisions.

Step 3. Fuzzification: Fuzzy sets, denoted as X_k, $k = 1, 2, 3,\ldots, m$, are assigned to each partition interval in the form of a triangular fuzzy number. These fuzzy numbers are characterized as the lower limit, midpoint, and upper limit. In the context of the actual dataset. This process aims to fuzzify the dataset, transforming it into an initial series.

Step 4. Defuzzification: The fuzzy set values are converted into crisp values by (2):

$$t_j = \begin{cases} \frac{1+0.5}{\frac{1}{a_1}+\frac{0.5}{a_2}} & j = 1 \\ \frac{0.5+1+0.5}{\frac{0.5}{a_{j-1}}+\frac{1}{a_j}+\frac{0.5}{a_{j+1}}} & 2 \le j \le m-1 \\ \frac{0.5+1}{\frac{0.5}{a_{j-1}}+\frac{1}{a_j}} & j = m \end{cases} \tag{2}$$

where $a_{j\text{-}1}$, a_j, and a_{j+1} are the midpoints of the interval of fuzzy sets $X_{k\text{-}1}$, X_k, and X_{k+1} carriers respectively.

Step 5. Model Application: The proposed model is applied as in (3):

$$\underset{i}{\hat{y}} = y_{i-1}\left(1 + \frac{t_j}{100}\right),\ i = 2, 3, 4, \ldots, n \tag{3}$$

As in [33], the forecasting outcomes obtained by Eq. (3) are anticipated to exhibit a higher accuracy rate compared to other FTS models and conventional time series models. Nevertheless, further enhancement in accuracy can be achieved by optimizing the parameters t_j, as opposed to using predetermined values as in Eq. (2). In this study, we have implemented this approach, and a detailed discussion of the results will follow.

Case study: PKR-US $ exchange rate forecasting using FTS

The FOREX significantly impacts Pakistan's economy, given its reliance on international trade, particularly with the US $. Fluctuations in FOREX have substantial effects on Pakistan's underdeveloped economy. A comprehensive dataset of the ER (1960–2021) is graphically depicted in Fig. 1.

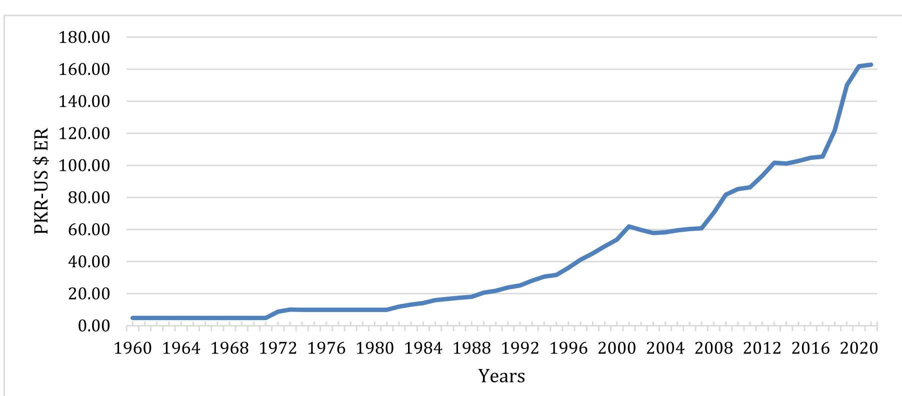

Fig. 1 PKR performance against US $ from 1960 to 2021

Table 1 Year-to-year actual Pak US $ ER and corresponding growth rate (T)

Years	PKR-US$	T	Years	PKR-US$	T	Years	PKR-US $	T
1960	4.7619	–	1981	9.9	0	2002	59.7238	− 3.558
1961	4.7619	0	1982	11.8475	19.6714	2003	57.752	− 3.3015
1962	4.7619	0	1983	13.117	10.7154	2004	58.2579	0.8759
1963	4.7619	0	1984	14.0463	7.0852	2005	59.5145	2.157
1964	4.7619	0	1985	15.9284	13.3989	2006	60.2713	1.2717
1965	4.7619	0	1986	16.6475	4.5147	2007	60.7385	0.7751
1966	4.7619	0	1987	17.3988	4.5129	2008	70.408	15.9199
1967	4.7619	0	1988	18.0033	3.4743	2009	81.7129	16.0562
1968	4.7619	0	1989	20.5415	14.0985	2010	85.1938	4.2599
1969	4.7619	0	1990	21.7074	5.6757	2011	86.3434	1.3494
1970	4.7619	0	1991	23.8008	9.6437	2012	93.3952	8.1672
1971	4.7619	0	1992	25.0828	5.3865	2013	101.629	8.816
1972	8.68138	82.3092	1993	28.1072	12.0576	2014	101.1	− 0.5203
1973	9.99425	15.1228	1994	30.5666	8.7501	2015	102.769	1.651
1974	9.9	− 0.943	1995	31.6427	3.5205	2016	104.769	1.946
1975	9.9	0	1996	36.0787	14.019	2017	2017	0.6548
1976	9.9	0	1997	41.1115	13.9496	2018	121.824	15.5221
1977	9.9	0	1998	45.0467	9.5719	2019	150.036	23.1581
1978	9.9	0	1999	49.5007	9.8876	2020	161.839	7.8662
1979	9.9	0	2000	53.6482	8.3787	2021	162.851	0.6258
1980	9.9	0	2001	61.9272	15.432			

In Fig. 1, the World Bank's annual exchange has been presented for a clearer insight into the case data from 1961 to 2021. We can see that exchange rates were recorded as 4.76 and the maximum as 162.85. By (1), we get the growth rate (T) as under in Table 1.

In Table 1, the Growth Rate reveals a range from − 3.56 to 82.31 for min. and max. values of T. Establishing a universe of discourse as U = [− 3.56, 82.31] via (2). Notably, the larger classes, such as the 1st and 2nd, are further subdivided into 20 and 10 sub-intervals, respectively. The fuzzification process involves applying fuzzy sets X_i, where $i = 1,2,3,\ldots,35$. Utilizing (3), our proposed model (4) is fitted to defuzzified data, as detailed in Table 2. A graphical summary is depicted in Fig. 2, encompassing all methods with specified parameters.

The AFER% demonstrates the superior performance of Fuzzy Forecasting, with a minimal result of 0.80%, surpassing the two-year moving average (4.7041%) and three-year moving average (9.2367%). The % AFER underscores the overall dominance of FTS forecasting compared to conventional time series methods. The forecast for the annual 2022 value using Eq. (3) and the last fuzzy set definition, X_7, yields an estimated value of 160 ER, with an interval estimation of (118.7716, 203.6467) against the full dataset. This interval estimation for the annual 2022 is poised to have significant consequences on Pakistan's economy, potentially influencing it either positively or negatively.

To enhance the forecasting accuracy of Eq. (3), adjustments can be made by optimizing parameters by Eq. (2), as opposed to utilizing specified values such as 0.5 for all parameters. The parameters γ_1, γ_2, δ_1 and δ_2 are subject to optimization for this purpose. The adjusted defuzzified t_j^{adj} is provided as follows:

Table 2 Forecasting results for PKR-US $ Exchange Rate

Years	y_i	T	X_k	t_j	$y_i^{\wedge}$	AEFR	Years	y_i	T	X_k	t_j	$y_i^{\wedge}$	AEFR
1960	4.76	–	–	–	–	–	1992	25.08	5.39	X15	5.47	25.1019	0.0013
1961	4.76	0	X6	− 0.26	4.7497	0.0043	1993	28.11	12.06	X23	12	28.0927	0.0009
1962	4.76	0	X6	− 0.26	4.7497	0.0043	1994	30.57	8.75	X20	8.67	30.5443	0.0012
1963	4.76	0	X6	− 0.26	4.7497	0.0043	1995	31.64	3.52	X12	3.57	31.6587	0.0008
1964	4.76	0	X6	− 0.26	4.7497	0.0043	1996	36.08	14.02	X25	14.51	36.2344	0.0072
1965	4.76	0	X6	− 0.26	4.7497	0.0043	1997	41.11	13.95	X25	14.51	41.3141	0.0082
1966	4.76	0	X6	− 0.26	4.7497	0.0043	1998	45.05	9.57	X21	9.58	45.0507	0.0001
1967	4.76	0	X6	− 0.26	4.7497	0.0043	1999	49.5	9.89	X21	9.58	49.3628	0.0046
1968	4.76	0	X6	− 0.26	4.7497	0.0043	2000	53.65	8.38	X20	8.67	53.7927	0.0045
1969	4.76	0	X6	− 0.26	4.7497	0.0043	2001	61.93	15.43	X26	15.77	62.1061	0.0048
1970	4.76	0	X6	− 0.26	4.7497	0.0043	2002	59.72	− 3.56	X1	− 3.01	60.0641	0.0095
1971	4.76	0	X6	− 0.26	4.7497	0.0043	2003	57.75	− 3.3	X1	− 3.01	57.927	0.0051
1972	8.68	82.31	X35	73.02	8.239	0.0849	2004	58.26	0.88	X7	− 1.21	57.0512	0.0345
1973	9.99	15.12	X25	14.51	9.9412	0.0089	2005	59.51	2.16	X10	2.29	59.5934	0.0022
1974	9.9	− 0.94	X5	− 0.35	9.9597	0.01	2006	60.27	1.27	X8	0.92	60.0637	0.0057
1975	9.9	0	X6	− 0.26	9.8747	0.0043	2007	60.74	0.78	X7	− 1.21	59.5399	0.0329
1976	9.9	0	X6	− 0.26	9.8747	0.0043	2008	70.41	15.92	X26	15.77	70.3142	0.0022
1977	9.9	0	X6	− 0.26	9.8747	0.0043	2009	81.71	16.06	X26	15.77	81.5082	0.0042
1978	9.9	0	X6	− 0.26	9.8747	0.0043	2010	85.19	4.26	X13	4.21	85.1498	0.0009
1979	9.9	0	X6	− 0.26	9.8747	0.0043	2011	86.34	1.35	X8	0.92	85.98	0.007
1980	9.9	0	X6	− 0.26	9.8747	0.0043	2012	93.4	8.17	X19	7.98	93.2319	0.0029
1981	9.9	0	X6	− 0.26	9.8747	0.0043	2013	101.63	8.82	X20	8.67	101.4932	0.0022
1982	11.85	19.67	X29	19.52	11.833	0.002	2014	101.1	-0.52	X5	− 0.35	101.2775	0.0029
1983	13.12	10.72	X22	10.74	13.1202	0.0004	2015	102.77	1.65	X9	1.63	102.7515	0.0003
1984	14.05	7.09	X18	7.35	14.0812	0.0041	2016	104.77	1.95	X9	1.63	104.448	0.0051
1985	15.93	13.4	X24	13.26	15.9083	0.0021	2017	105.46	0.65	X7	− 1.21	103.4977	0.0309

Table 2 (continued)

Years	y_i	T	X_k	t_j	$y_i^{\wedge}$	AEFR	Years	y_i	T	X_k	t_j	$y_i^{\wedge}$	AEFR
1986	16.65	4.51	X13	4.21	16.5984	0.0049	2018	121.82	15.52	X26	15.77	122.0806	0.0035
1987	17.4	4.51	X13	4.21	17.3477	0.0049	2019	150.04	23.16	X31	27.56	155.3954	0.0595
1988	18	3.47	X12	3.57	18.0204	0.0016	2020	161.84	7.87	X19	7.98	162.0062	0.0017
1989	20.54	14.1	X25	14.51	20.6158	0.006	2021	162.85	0.63	X7	− 1.21	159.8746	0.0305
1990	21.71	5.68	X15	5.47	21.6644	0.0033	Total						0.4882
1991	23.8	9.64	X21	9.58	23.7873	0.0009	%AFER	`					0.80%

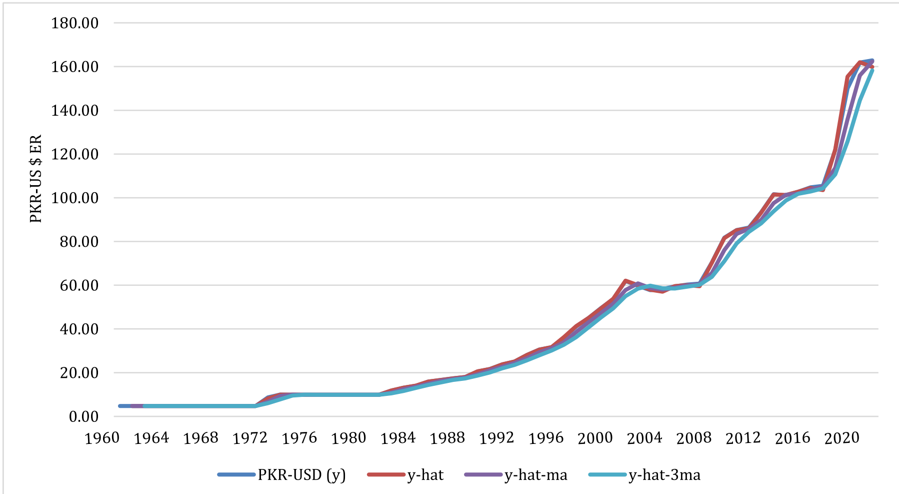

Fig. 2 Performance of PKR-US $ Actual data, fuzzy forecasting, two and three years moving average time series models

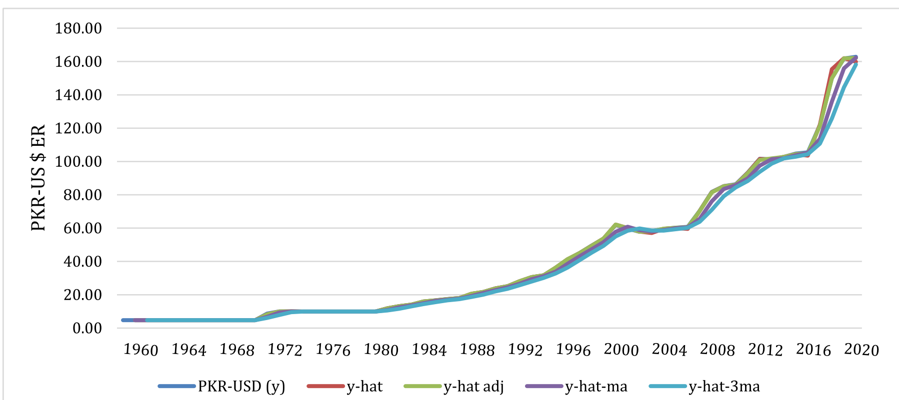

Fig. 3 Performance Summary of PKR-US $ Actual Data, Fuzzy Forecasting, Adjusted Fuzzy Forecasting, Two and Three Years Moving Average Time Series Model

$$t_j^{adj} = \begin{cases} \frac{1+\gamma_1}{\frac{1}{a_1}+\frac{\gamma_1}{a_2}} & j = 1 \\ \frac{\delta_1+1+\delta_2}{\frac{\delta_1}{a_{j-1}}+\frac{1}{a_j}+\frac{\delta_2}{a_{j+1}}} & 2 \le j \le m-1 \\ \frac{\gamma_2+1}{\frac{\gamma_2}{a_{j-1}}+\frac{1}{a_j}} & j = m \end{cases} \tag{4}$$

For optimization of these parameters, simultaneous equation systems (SES) versus partial differential equations (PDEs) were utilized. We get estimates under SES as $\gamma_1 = 0.5011, \gamma_2 = 0.5005$, $\delta_1 = -\ 0.1239$ and $\delta_2 = -\ 0.4221$ by using Maple 17. It is pertinent to mention that we got $\delta_1 + \delta_2 = -1$ as $\delta_1 = -\ 0.5406$ and $\delta_2 = -\ 0.4594$ by differential equation optimization method which vanishes t_j^{adj} as $t_j^{adj} = 0$ for all $2 \le j \le m-1$. So, in this study optimization of parameters through SES was successful only. AFER under (4) through (3) was found 34.69% which is much better than the model with specified

Table 3 ARIMA (0,2,2) Summary

Model	Coefficient	Std. Error	z
Constant	0.132755	0.0413627	3.21
ϕ_1	− 0.261625	0.0892098	− 2.933
ϕ_2	− 0.738375	0.082883	− 8.909

Table 4 ARIMA (0,2,2) Summary

Mean dependent Var	0.016881	S.D. Dependent	4.577102
Mean of Innovations	− 0.171839	S.D. Of Innovations	3.340195
R-Squared	0.99386	Adjusted R-Squared	0.793754
Log-Likelihood	− 159.4014	Akaike Criterion	326.8028
Schwarz Criterion	335.1802	AFER	0.830797

Table 5 ARIMA (0,2,2) model forecast

Years	Prediction	Std. Error	95% C.I
2022	167.911	3.3402	(161.365, 174.458)
2023	174.806	6.69869	(161.677, 187.935)
2024	181.834	8.86499	(164.459, 199.209)
2025	188.994	10.5973	(168.224, 209.765)
2026	196.287	12.0838	(172.603, 219.971)

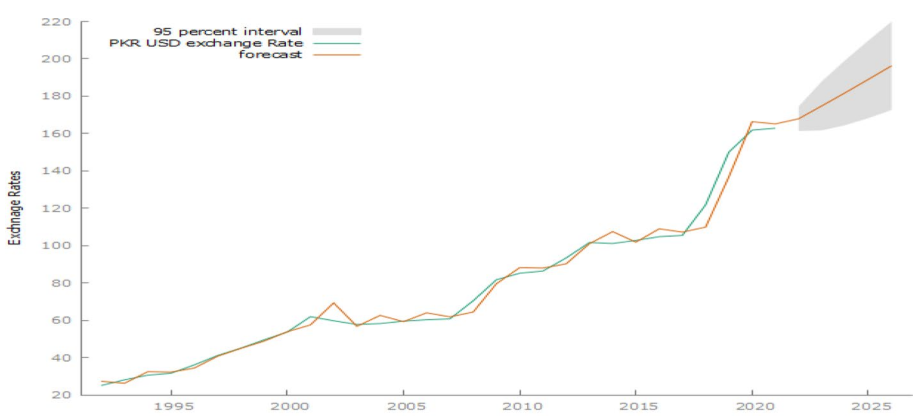

Fig. 4 Actual Data Versus ARIMA (0, 2, 2) Plotting and Forecasting

values. The projected forecast for the annual 2022 value using Eq. (4) and the last fuzzy set definition X_7 yields an estimated value of 163.1328 ER, with an interval estimation of (120.4759, 205.7897) against the complete dataset. The potential impact of this interval estimation is expected to be significant for Pakistan's dynamic economy in 2022. Figure 3 illustrates the performance of all methods with optimized parameters under Eq. (4).

Using Gretl, we identified the ARIMA (0,2,2) model through the AIC and BIC model selection criteria, showcasing highly significant parameters. The detailed results are

Table 6 Overall comparison of all models

Model	AFER (%)	Forecast for 2022	95% C.I	Relative efficiency
Specified Parameters SP	0.9412	158.6337	117.8123, 202.3021	100
Proposed	0.8046	160	118.7716, 203.6467	116.9774
Adjusted Parameters SP	0.8791	161.2916	119.1294, 201.7119	107.064
Proposed	0.5227	163.1328	120.4759, 205.7897	180.065
Two Years Moving Average	4.7041	157.493	122.1825, 196.1081	20.00808
Three Years Moving Average	9.2367	156.2387	122.9123, 199.2912	10.18979
ARIMA	0.8308	167.911	161.3650, 174.4580	113.2884

presented in Tables 3, 4, and 5, with Fig. 4 providing a visual representation of the outcomes.

Table 5 and Fig. 4, presenting the forecast results for the ARIMA (0,2,2) model over the next 5 years, clearly depict a consistent and gradual upward trajectory in forex rates. This trend suggests a sustained increase in FOREX, emphasizing potential economic shifts.

The comparative forecasting analysis of all models under study, presented in Table 6, indicates that the fuzzy proposed model surpasses all specified competitive class models and those with optimized parameters, achieving the lowest Absolute Forecasting Error Rate (AFER). The relative efficiency for this case outperforms all methods. Figures 2 and 3 visually demonstrate that the proposed models closely align with actual data points, showcasing superior performance compared to all studied methods.

Results

The extensive and transformative revisions implemented in Model [33], coupled with the meticulous examination of PKR-US $ data, have led to a groundbreaking discovery: the utilization of Fuzzy Time Series (FTS) incorporating triangular fuzzy numbers results in achieving the lowest Absolute Forecasting Error Rate (AFER) when juxtaposed with a diverse array of traditional time series methods. This remarkable achievement not only serves as a testament to the robustness and efficacy of the FTS approach but also underscores its unparalleled effectiveness in capturing the intricate nuances of exchange rate dynamics with remarkable precision and granularity.

Beyond its exceptional performance, what sets the FTS methodology apart is its inherent simplicity and versatility, which not only facilitates its implementation but also invites further exploration and refinement. This inherent simplicity not only broadens the accessibility of the methodology but also lays the groundwork for future research endeavors aimed at advancing FTS methodologies through the exploration of a wider spectrum of fuzzy numbers, the incorporation of additional parameters, and the development of sophisticated algorithms to enhance predictive accuracy and reliability.

Moreover, the enhanced Model [33], enriched with interval estimation techniques, introduces a revolutionary paradigm shift in the landscape of exchange rate forecasting. By integrating interval estimation, the model not only elevates the precision and reliability of predictions but also provides a comprehensive and nuanced understanding of future trends and uncertainties, thus empowering stakeholders with invaluable insights for strategic decision-making and risk management.

This groundbreaking approach not only propels the field of exchange rate forecasting to new heights but also carries profound implications for various sectors of the economy. As stakeholders harness the power of advanced forecasting methodologies, they gain a competitive advantage in navigating the intricate web of global markets, thereby fostering sustainable growth, resilience, and prosperity on both national and international scales, paving the way for a more robust and resilient economic future.

Discussion

This groundbreaking and forward-thinking approach, characterized by its innovative provision of short-term predictions for the year 2022 using interval estimation, holds profound significance for businesses nationwide. The transformative potential of these meticulously derived insights transcends the realms of academia, offering an invaluable compass for decision-makers across diverse industries throughout the country. By providing nuanced forecasts that meticulously account for uncertainty, this research empowers businesses with an arsenal of indispensable tools needed to navigate the complexities of volatile economic landscapes with unwavering confidence and agility.

Furthermore, the practical implications of this study reverberate deeply within the fabric of the business community, where astute and informed decision-making is not merely a necessity but a cornerstone of sustained success and growth. By offering actionable insights into the intricate dynamics of exchange rates and their potential ripple effects, this research enables businesses to proactively adapt their strategies, optimize resource allocation, and capitalize on emerging opportunities with precision and efficacy unparalleled. The integration of interval estimation into the forecasting model not only elevates the accuracy of predictions but also furnishes decision-makers with a more comprehensive understanding of potential fluctuations and risks, thus facilitating the formulation of robust contingency plans and comprehensive risk management strategies.

The study stems from the need to improve the accuracy of exchange rate forecasts, especially in the context of volatile currency markets like the PKR-USD exchange rate. Traditional models often fail to capture the intricate dynamics of exchange rates, leading to less reliable predictions. By introducing Fuzzy Time Series (FTS) with triangular fuzzy numbers and advanced partitioning techniques, our study addresses this gap, achieving significantly lower forecasting error rates. This approach enhances the precision and reliability of predictions, making it a valuable tool for both researchers and practitioners. In terms of policy implications, the study's findings hold significant relevance for central banks, businesses, and governments. For monetary authorities, the improved forecasting model can be used to inform decisions related to currency interventions, interest rate adjustments, and inflation targeting. Accurate forecasts enable policymakers to better manage exchange rate volatility, thus stabilizing the economy. For businesses, particularly those involved in international trade, the model provides a more reliable basis for currency risk management and strategic planning, and helping firms mitigate financial risks and optimize resource allocation. Additionally, for economic planners, the FTS model offers valuable insights into future trends, enabling more effective contingency planning and long-term economic strategies.

In essence, this research not only enriches the academic discourse but also serves as a beacon of practical utility, epitomizing the pinnacle of forecasting advancements in economic analysis. By seamlessly bridging theoretical insights with real-world application, it not only fosters a deeper comprehension of market dynamics but also empowers businesses with the indispensable knowledge and foresight needed to thrive in an ever-evolving economic landscape. As such, the insights gleaned from this study are poised to profoundly inform strategic decision-making processes, inspire innovation, and ultimately, propel the overall economic prosperity and competitive edge of the nation to unprecedented heights.

Conclusions

The research presented in this study represents a deliberate endeavor to elevate the reliability of future estimates, aiming for a minimal Absolute Forecasting Error Rate (AFER) compared to competing models. Our exploration has led us to a key determinant of forecasting accuracy—the partitioning of the UoD.

In contrast to the conventional sub-partitioning rule of 4–3–2 proposed by [33], our study advocates for a nuanced and adaptable approach, adopting a 20–10 sub-partitioning rule. This tailored strategy has proven effective in achieving targeted results, reinforcing the importance of customizing partitioning strategies to align with the unique characteristics of each study.

A noteworthy revelation from our findings is the positive correlation between finer UoD partitioning and heightened forecast accuracy. This not only challenges established norms but also underscores the critical role of customization in partitioning strategies, recognizing that a one-size-fits-all approach may not universally apply.

Within the current scholarly landscape, a conspicuous void exists in comprehensive studies focusing on multiple step-ahead forecasts within the realm of FTS. Concurrently, the ongoing quest for an optimal optimization method adds a layer of complexity and intrigue to the research landscape, beckoning further exploration and advancement.

In summary, FTS methods present an evolving frontier of research, offering invaluable insights into the nuanced modeling of economic indicators for the holistic betterment of the country. The continuous refinement of forecasting methodologies, driven by sophisticated data analytics, holds the promise of providing decision-makers with nuanced and precise information, contributing to the formulation of more astute and effective economic policies.

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Author contributions
M.B: Conceptualization, data curation, investigation, methodology, M.A.: writing original draft, writing review, and editing. S.A.: supervised and administrated the article writing work, Necessary resources required to complete the article, S.M.N.: software, analysis, simulations; O.A.A.: Computed data, software, methodology, writing revision, M.E.A.E.: Data analysis,

analyzed data in tables, software, revised methodology, writing revision; I.K.: methodology. plots, numerical analysis, discussion. All authors read and approved the final manuscript.

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Declarations

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"Not applicable." For the current work, the authors have not conducted any animal or human trials.

Consent for publication
Not applicable. Individual data or information has not been included by the authors for the current work.

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