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Analysis of Graeco-Latin square designs in the presence of uncertain data

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Abstract

Objective: This paper addresses the Graeco-Latin square design (GLSD) under neutrosophic statistics. In this work, we propose a novel approach for analyzing Graeco-Latin square designs using uncertain observations.

Method: This approach involves the determination of a neutrosophic ANOVA and the determination of the neutrosophic hypotheses and decision rule.

Results: The performance of the proposed design is evaluated using the numerical examples and simulation study.

Conclusion: Based on the results observed, it can be concluded that the GLSD under neutrosophic statistics performs better than the GLSD under classical statistics in the presence of uncertainty.

Keywords: Neutrosophic statistics, Uncertain observations, GLSD, Neutrosophic hypotheses

Introduction

Latin square designs are among the most frequently used experimental designs. In this design, each treatment occurs once, and only once, in each row and column; thus, the number of treatments, rows, and columns is equal. The GLSD is another design related to the Latin square. The Graeco-Latin square consists of two orthogonal Latin squares (each letter combination appears exactly once). A GLSD allows us to investigate up to four factors within a single design. The two factors are represented in rows and columns, while the two others represent in Latin and Greek letters. A Graeco-Latin square was first constructed by Euler, Leonhard in 1782. Yates and Mather [1] provided Graeco-Latin tables of orders 3 to 12 (excluding the order of six). A comprehensive description of GLSDs was also included in Dénes and Keedwell [2]. Dodge and Shah [3] addressed the estimation of missing data in Latin squares and Graeco-Latin squares. Preece [4] discussed non-orthogonal GLSDs. Street [5] used the theory of cyclotomy to construct certain balanced incomplete block designs (BIBDs) and partially balanced incomplete block designs (PBIBDs), which gave some GLSDs as well as some nested row and column designs. Seberry [6] highlighted orthogonal GLSDs. You can find related articles and books about GLSD in [7–11].

Neutrosophic logic is claimed by Smarandache [12] to be more efficient than fuzzy logic. Smarandache [13] introduced the concept of neutrosophic statistics (NS), an extension of classical statistics. Aslam [14] explained the differences between fuzzy statistics, NS, and classical statistics. Neutrosophic ANOVA has been highlighted by Aslam [15]. In a more recent article, AlAita and Aslam [16] discussed the application of neutrosophic analysis of covariance to neutrosophic completely random designs, neutrosophic randomized complete block designs, and neutrosophic split-plot designs. Aslam and Albassam [17] suggested post-hoc multiple comparison tests under NS. Neutrosophic correlation and simple linear regression have been discussed by Salama, Khaled [18]. Analysis of neutrosophic multiple regression has been suggested by Nagarajan, Broumi [19]. Numerous neutrosophic statistical studies have been discussed in [20–27].

The GLSD is available under classical statistics in the literature. However, the test statistics of this design are not capable of providing information regarding the measure of indeterminacy under uncertainty. The main objective of the study is to solve problems associated with studies involving imprecise, vague, and uncertain data that require the application of Graeco-Latin square designs. We can, therefore, analyze our proposed designs using NS in order to provide additional information on the measure of indeterminacy that classical statistics are not able to provide. There are many real-world examples that enable us to use this design under NS, for example, a study on the differential growth of some algae in acetic acid. For this study, the 5×5 GLSD is used. In which, five different algae were being studied in five different types of vessels with five settings of the pH level. Greek letters represent temperatures at five levels, columns represent pH settings, rows represent vessels, and Latin letters represent algae types. In this example, classical statistics cannot analyze and interpret neutrosophic data (where the data has some degree of indeterminacy) for GLSD. Therefore, the test we propose for neutrosophic Graeco-Latin square design (NGLSD) in this paper are essential for these studies.

According to a literature review, no work has been conducted on GLSDs under NS. In this paper, we propose for the first time a NGLSD. Moreover, an NANOVA Table will be organized to determine the proposed F_N-test, neutrosophic hypotheses, and decision rule. A Numerical examples and simulation study we will conduct to evaluate the performance of the proposed design F_N-test. Our expectation is that the proposed Greek-Latin square design will perform better than the existing design in the event of uncertainty.

Preliminaries

Many articles and books have been published recently that use NS, which is a generalization of classical statistics. NS is also characterized by its flexibility and efficiency in uncertain environments, as well as the ability to calculate measure of indeterminacy resulting from the state of uncertainty. It is possible to categorize these uncertainties into three categories: Degree of Truth (T), Degree of Falsehood (F) and Degree of Indeterminacy (I). Below is a brief overview of some basic concepts related to NS.

Suppose that a neutrosophic random variable (NRV) $X_N \in [X_L, X_U]$ follows the neutrosophic normal distribution (NND) with a neutrosophic population mean $\mu_N \in [\mu_L, \mu_U]$ and a neutrosophic population variance $\sigma_N^2 \in [\sigma_L^2, \sigma_U^2]$, where X_L and X_U are smaller and larger values of indeterminacy interval. Let $X_N = X_L + X_U I_N$ is the neutrosophic form of NRV having determinate part X_L and indeterminate part $X_U I_N$; $I_N \in [I_L, I_U]$, where $I_N \in [I_L, I_U]$ is indeterminate interval.

Suppose $n_N \in [n_L, n_U]$ is a neutrosophic random sample selected from a population of size N_N having indeterminate observations. The neutrosophic population means μ_N and variance σ_N^2, are expressed as follows;

$\mu_N \in \left[\frac{\sum_{i=1}^{N_L} X_{Li}}{N_L}, \frac{\sum_{i=1}^{N_U} X_{Ui}}{N_U}\right]$; $\mu_N \in [\mu_L, \mu_U]$ and $\sigma_N^2 \in \left[\frac{\sum_{i=1}^{N_L} (X_{Li}-\mu_L)^2}{N_L}, \frac{\sum_{i=1}^{N_U} (X_{Ui}-\mu_U)^2}{N_U}\right]$; $\sigma_N^2 \in [\sigma_L^2, \sigma_U^2]$.

But, in the numerical examples, μ_N and σ_N^2 are unknown and can be estimated using the sample observations. The neutrosophic sample mean $\overline{X}_N$ and the variance s_N^2, are expressed by;

$\overline{X}_N \in \left[\frac{\sum_{i=1}^{n_L} X_{Li}}{n_L}, \frac{\sum_{i=1}^{n_U} X_{Ui}}{n_U}\right]$; $\overline{X}_N \in \left[\overline{X}_L, \overline{X}_U\right]$ and $s_N^2 \in \left[\frac{\sum_{i=1}^{n_L} (X_{Li}-\overline{X}_L)^2}{n_L-1}, \frac{\sum_{i=1}^{n_U} (X_{Ui}-\overline{X}_U)^2}{n_U-1}\right]$; $s_N^2 \in [s_L^2, s_U^2]$.

Analysis of neutrosophic Graeco-Latin square design

Model and NANOVA for a neutrosophic Graeco-Latin square design

The neutrosophic statistical model for a NGLSD with a_N rows and b_N columns can be expressed as:

$$y_{Nijkl} = \mu_N + \omega_{Ni} + \tau_{Nj} + \gamma_{Nk} + \delta_{Nl} + \varepsilon_{Nijkl}; \begin{cases} i = 1,2,\ldots,p_N \\ j = 1,2,\ldots,p_N \\ k = 1,2,\ldots,p_N \\ l = 1,2,\ldots,p_N \end{cases}, \tag{2}$$

The neutrosophic form of y_{Nijkl} can be expressed as

$$y_{Nijkl} = y_{Nijkl} + y_{Nijkl} I_N; I_N \in [I_L, I_U], \tag{3}$$

where y_{Nhqi} represents the neutrosophic observation in the ith row and kth column for Latin letter j and Greek letter k, μ_N represents a neutrosophic overall mean, ω_{Ni} represents the neutrosophic effect of the ith row, τ_{Nj} represents the neutrosophic effect of the jth treatment of the Latin letter, γ_{Nk} represents the neutrosophic effect of the kth treatment of the Greek letter, δ_{Nl} represents the neutrosophic effect of the lth column, and ε_{Nijkl} represents the neutrosophic random error assumed to have mean of zero and variance $\sigma_N^2 \in [\sigma_L^2, \sigma_U^2]$. Let the total neutrosophic number of all plots in the rows and columns is n_{NT}; then $n_{NT} = p_N^2$. Table 1 presents NANOVA of NGLSD.

NSSs can be computed using the following formulas:

$SS_{NT} = \sum_{i=1}^{p_N} \sum_{j=1}^{p_N} \sum_{k=1}^{p_N} \sum_{l=1}^{p_N} y_{Nijkl}^2 - \frac{y_{N....}^2}{p_N^2}$; $SS_{NT} \in [SS_{LT}, SS_{UT}]$,

$SS_{NR} = \frac{1}{p_N} \sum_{i=1}^{p_N} y_{Ni...}^2 - \frac{y_{N....}^2}{p_N^2}$; $SS_{NR} \in [SS_{LR}, SS_{UR}]$,

$SS_{NL} = \frac{1}{p_N} \sum_{j=1}^{p_N} y_{N.j..}^2 - \frac{y_{N....}^2}{p_N^2}$; $SS_{NR} \in [SS_{LL}, SS_{UL}]$,

$SS_{NG} = \frac{1}{p_N} \sum_{k=1}^{p_N} y_{N..k.}^2 - \frac{y_{N....}^2}{p_N^2}$; $SS_{Nk} \in [SS_{LG}, SS_{UG}]$,

$SS_{NC} = \frac{1}{p_N} \sum_{l=1}^{p_N} y_{N...l}^2 - \frac{y_{N....}^2}{p_N^2}$; $SS_{NR} \in [SS_{LC}, SS_{UC}]$,

Table 1 NANOVA Table for NGLSD

Source	NSS	ndf	NMS	F_N
Rows	SS_{NR}	p_N-1	$MS_{NR}=\frac{SS_{NR}}{p_N-1}$	$F_{NR}=\frac{MS_{NR}}{MS_{NE}}$
Latin letter treatments	SS_{NL}	p_N-1	$MS_{NL}=\frac{SS_{NL}}{p_N-1}$	$F_{NL}=\frac{MS_{NL}}{MS_{NE}}$
Greek letter treatments	SS_{NG}	p_N-1	$MS_{NG}=\frac{SS_{NG}}{p_N-1}$	$F_{NG}=\frac{MS_{NG}}{MS_{NE}}$
Columns	SS_{NC}	p_N-1	$MS_{NC}=\frac{SS_{NC}}{p_N-1}$	$F_{NC}=\frac{MS_{NC}}{MS_{NE}}$
Error	SS_{NE}	$(p_N-3)(p_N-1)$	$MS_{NE}=\frac{SS_{NE}}{(p_N-3)(p_N-1)}$	
Total	SS_{NT}	p_N^2-1		

$$SS_{NE}=SS_{NT}-SS_{NR}-SS_{NL}-SS_{NG}-SS_{NC};SS_{NE}\in[SS_{LE},SS_{UE}],$$

where $y_{Ni...}$ stands for the total number of the neutrosophic observations in the hth neutrosophic row, $y_{N.j..}$ stands for the total number of the neutrosophic observations in the jth neutrosophic treatment of the Latin letter, $y_{N..k.}$ stands for the total number of the neutrosophic observations in the kth neutrosophic treatment of the Greek letter, $y_{N...l}$ stands for the total number of the neutrosophic observations in the lth neutrosophic column, and $y_{N....}$ stands for the total number of all the neutrosophic observations.

Neutrosophic mean squares are defined as:
$MS_{NR}=\frac{SS_{NR}}{p_N-1}; MS_{NR}\in[MS_{LR},MS_{UR}]$, $MS_{NL}=\frac{SS_{NL}}{p_N-1}; MS_{NL}\in[MS_{LL},MS_{UL}]$,
$MS_{NG}=\frac{SS_{NG}}{p_N-1}; MS_{NG}\in[MS_{LG},MS_{UG}]$, $MS_{NC}=\frac{SS_{NC}}{p_N-1}; MS_{NC}\in[MS_{LC},MS_{UC}]$,
$MS_{NE}=\frac{SS_{NE}}{(p_N-3)(p_N-1)}; MS_{NE}\in[MS_{LE},MS_{UE}]$.

The neutrosophic statistic F_N-tests become
$F_{NR}=\frac{MS_{NR}}{MS_{NE}};\quad F_{NR}\in[F_{LR},F_{UR}]$, $F_{NL}=\frac{MS_{NL}}{MS_{NE}};\quad F_{NL}\in[F_{LL},F_{UL}]$, $F_{NG}=\frac{MS_{NG}}{MS_{NE}}$; $F_{NG}\in[F_{LG},F_{UG}]$, $F_{NC}=\frac{MS_{NC}}{MS_{NE}}; F_{NC}\in[F_{LC},F_{UC}]$.

The neutrosophic form of F_N is:

$$F_N=F_L+F_UI_{F_N};I_{F_N}\in\left[I_{F_L},I_{F_U}\right],$$

where F_L and $F_UI_{F_N}$ are determinate and indeterminate parts of each the proposed test. This test reduces to test under classical statistic if $I_{F_N}=0$.

Neutrosophic hypotheses and decision rule

Under neutrosophic statistics, a null hypothesis and an alternative hypothesis are presented as follows:

$$\begin{aligned}
&H_{N0}:\omega_{N1}=\omega_{N2}=\cdots=\omega_{Np}=0\quad vs\quad H_{N1}: at\ least\ one\ \omega_{Ni}\neq 0,\\
&H_{N0}:\tau_{N1}=\tau_{N2}=\cdots=\tau_{Np}=0\quad vs\quad H_{N1}: at\ least\ one\ \tau_{Nj}\neq 0,\\
&H_{N0}:\gamma_{N1}=\gamma_{N2}=\cdots=\gamma_{Np}=0\quad vs\quad H_{N1}: at\ least\ one\ \gamma_{Nk}\neq 0,\\
&H_{N0}:\delta_{N1}=\delta_{N2}=\cdots=\delta_{N3}=0\quad vs\quad H_{N1}: at\ least\ one\ \delta_{Nl}\neq 0.
\end{aligned}$$

The null hypothesis is accepted if $min\{p_N - value\} \geq \alpha$, where α is a level of significance. While, the null hypothesis is rejected if $max\{p_N - value\} < \alpha$.

Numerical examples and simulation study

Numerical examples

Example 4.1 Suppose that a researcher is investigating the effects of neutrosophic treatments on a particular study. The 4×4 NGLSD is used. The plan compares four neutrosophic treatments (Latin letters) in four neutrosophic rows, four neutrosophic columns, and four neutrosophic Greek letters. Table 2 summarizes the data.

In the NANOVA Table 3, we summarize the calculation formulas for testing the following null hypotheses against the alternative hypotheses under the neutrosophic statistics in NGLSD.

Example 4.2 In this example, the 5×5 NGLSD is used. The plan compares five neutrosophic treatments (Latin letters) in five neutrosophic rows, five neutrosophic columns, and five neutrosophic Greek letters. Table 4 summarizes the data.

Also, the results of Example 4.2 can be summarized in the NANOVA Table 5.

In order to conduct the proposed F_N-test for NGLSD, the following steps will need to be taken:

Step 1: We assign the neutrosophic test hypotheses.
Step 2: We prepare the NANOVA Table for the proposed design.
Step 3: We calculate the $p_N - value$ at the level of significance $\alpha = 0.05$. For example, from the NANOVA Table 3 in Example 4.1: $p_N - value = [0.030, 0.025]$.
Step 4: We accept the null hypothesis H_{N0} if $p_N - values \geq 0.05$, and we reject H_{N0} if $p_N - values < 0.05$.

In Table 3, we reject the null hypothesis H_{N0} because $p_N - value = [0.030, 0.025] < 0.05$. i.e., there is a difference in mean between the three treatments.

Table 2 Data for the NGLSD

Rows	Columns			
	1	**2**	**3**	**4**
1	$A\alpha$ [32.61, 33.49]	$B\beta$ [59.93, 60.24]	$C\gamma$ [45.64, 46.49]	$D\delta$ [61.59, 61.82]
2	$B\delta$ [56.01, 56.64]	$A\gamma$ [35.33, 35.66]	$D\beta$ [64.56, 65.13]	$C\alpha$ [42.20, 42.48]
3	$C\beta$ [51.08, 51.18]	$D\alpha$ [44.83, 45.76]	$A\delta$ [52.05, 52.18]	$B\gamma$ [51.62, 51.80]
4	$D\gamma$ [45.79, 46.56]	$C\delta$ [40.45, 40.82]	$B\alpha$ [59.62, 59.77]	$A\beta$ [49.53, 50.46]

Table 3 NANOVA Table for the NGLSD

Source	*NSS*	*ndf*	*NMS*	F_N	Neutrosophic form F_N	$p_N - value$
Rows	[3.07, 2.67]	[3,3]	[1.02, 0.89]	[0.069, 0.071]	$0.069 + 0.071 I_{F_{NR}}; I_{F_{NR}} \in [0, 0.028]$	[0.972, 0.972]
Latin letter treatments	[590.45, 584.66]	[3,3]	[196.82, 194.88]	[13.35, 15.56]	$13.35 + 15.56 I_{F_{NL}}; I_{F_{NL}} \in [0, 0.142]$	[0.030, 0.025]
Greek letter treatments	[404.19, 395.73]	[3,3]	[134.73, 131.91]	[9.14, 10.53]	$9.14 + 10.53 I_{F_{NG}}; I_{F_{NG}} \in [0, 0.132]$	[0.051, 0.042]
Columns	[269.78, 263.15]	[3,3]	[89.93, 87.72]	[6.10, 7.00]	$6.10 + 7.00 I_{F_{NC}}; I_{F_{NC}} \in [0, 0.129]$	[0.08 6, 0.072]
Error	[44.24, 37.57]	[3,3]	[14.75, 12.52]			
Total	[1311.73, 1283.78]	[15,15]				

Table 4 Data for the NGLSD

Rows	Columns				
	1	**2**	**3**	**4**	**5**
1	$A\alpha$ [10.03, 10.28]	$B\gamma$ [9.72, 10.32]	$C\epsilon$ [9.45, 9.92]	$D\beta$ [9.63, 10.24]	$E\delta$ [11.42, 11.70]
2	$B\beta$ [12.30, 12.78]	$C\delta$ [10.35, 10.63]	$D\alpha$ [12.03, 12.47]	$E\gamma$ [8.41, 9.08]	$A\epsilon$ [8.99, 9.63]
3	$C\gamma$ [12.71, 13.41]	$D\epsilon$ [11.55, 11.74]	$E\beta$ [11.56, 12.56]	$A\delta$ [10.53, 10.78]	$B\alpha$ [12.03, 12.64]
4	$D\delta$ [11.25, 12.03]	$E\alpha$ [7.37, 7.66]	$A\gamma$ [10.39, 11.24]	$B\epsilon$ [11.15, 11.39]	$C\beta$ [10.05, 10.96]
5	$E\epsilon$ [9.53, 10.27]	$A\beta$ [8.40, 8.44]	$B\delta$ [11.15, 11.99]	$C\alpha$ [9.29, 9.98]	$D\gamma$ [13.61, 13.99]

Simulation study

Simulated studies have been conducted in order to determine the effectiveness of the proposed F_N-test compared to the existing F-test.

Based on the empirical type I error rate and the power of the test $(1-\beta)$, we compare the proposed test and the existing test to determine the efficiency of our results. In this simulation study, different levels of significance α are considered (0.10, 0.05, 0.025, and 0.01).

We have selected the number of observations based on various previously published examples for Graeco-Latin square design (Montgomery [7]). The distribution considered in this study is the neutrosophic standard normal distribution and the number of Monte Carlo (MC) simulations is 10,000 replications.

The simulation study will be discussed in the following manner:

An MC method for calculating the empirical type I error rate using neutrosophic statistics involves the following steps:

- For every replicate, $u = 1,2, ..., a_N$:

  (a) Under the null hypothesis H_{N0}, generate the u th neutrosophic random sample $x_{N1}^{(u)}, x_{N2}^{(u)}, ..., x_{Nn}^{(u)}$.

  (b) Calculate the statistic F_{Nu}-test based on the u th sample.

  (c) Record the F_{Nu}-test results $I_{Nu} = 1$ if H_{N0} is rejected at the significance level α and accepted $I_{Nu} = 0$.

- Calculate the ratio of significant tests $\frac{1}{a_N}\sum_{u=1}^{a_N} I_{Nu}$. This ratio is the empirical type I error rate under the neutrosophic statistics. For further explanation, please see Fig. 1.

An MC method for calculating the empirical power of a test using neutrosophic statistics involves the following steps:

- Choose a particular value for the parameters. For example, $(\mu_{N1}, \mu_{N2}, \mu_{N3}, \mu_{N4}) = (0,1,1,1)$.
- For every replicate, $u = 1,2, ..., a_N$:

Table 5 NANOVA Table for the NGLSD

Source	*NSS*	*ndf*	*NMS*	F_N	Neutrosophic form F_N	$p_N - value$
Rows	[9.06, 9.40]	[4,4]	[2.26, 2.35]	[1.63, 2.36]	$1.63 + 2.36I_{F_{NR}}; I_{F_{NR}} \in [0, 0.309]$	[0.257, 0.141]
Latin letter treatments	[16.25, 16.40]	[4,4]	[4.06, 4.10]	[2.93, 4.11]	$2.93 + 4.11I_{F_{NL}}; I_{F_{NL}} \in [0, 0.287]$	[0.092, 0.042]
Greek letter treatments	[3.40, 4.32]	[4,4]	[0.85, 1.08]	[0.61, 1.08]	$0.61 + 1.08I_{F_{NG}}; I_{F_{NG}} \in [0, 0.435]$	[0.666, 0.426]
Columns	[13.31, 18.09]	[4,4]	[3.33, 4.52]	[2.40, 4.53]	$2.40 + 4.53I_{F_{NC}}; I_{F_{NC}} \in [0, 0.470]$	[0.136, 0.033]
Error	[11.11, 7.98]	[8,8]	[1.39, 1.00]			
Total	[53.13, 56.19]	[24,24]				

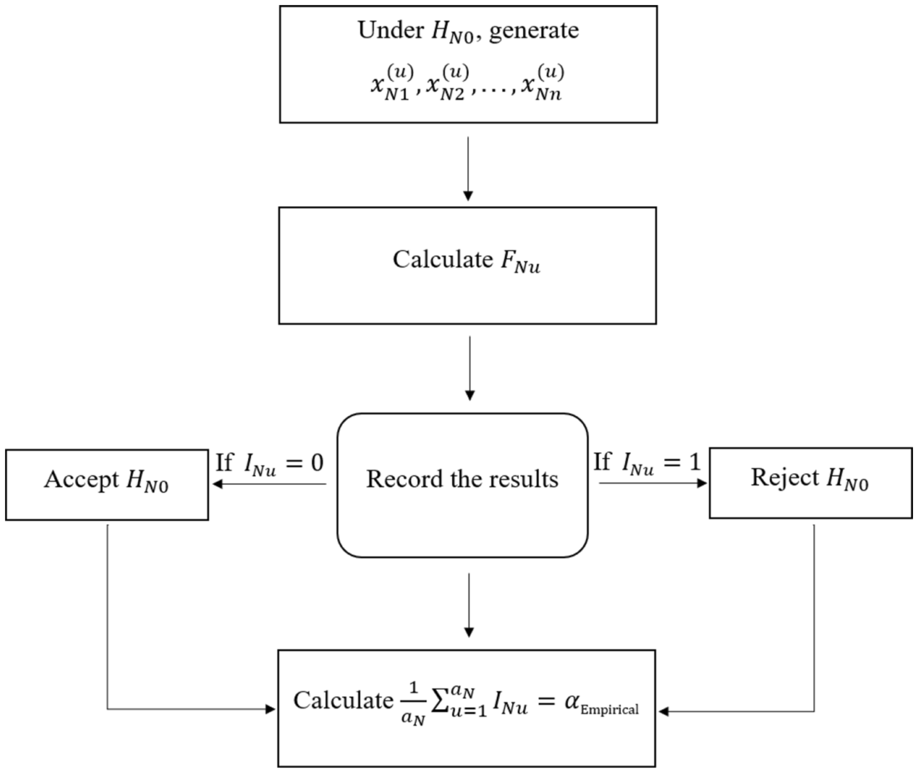

Fig. 1 MC simulation for calculating $\alpha_{Empirical}$

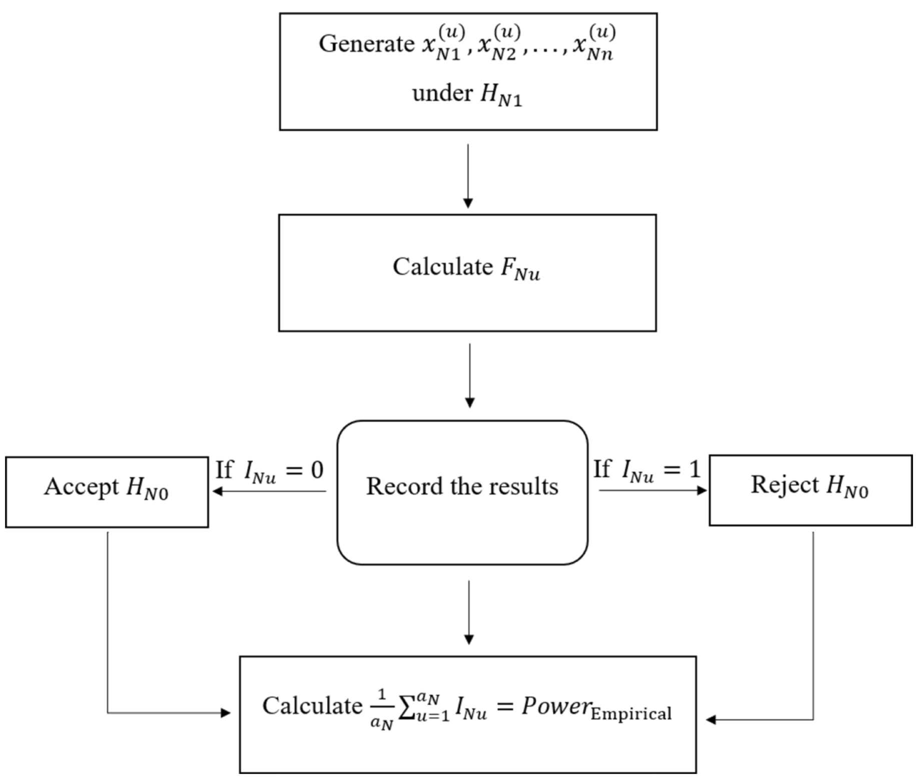

Fig. 2 MC simulation for calculating $Power_{Empirical}$

(a) Under the null hypothesis H_{N1}, generate the u th neutrosophic random sample $x_{N1}^{(u)}, x_{N2}^{(u)}, ..., x_{Nr}^{(u)}$.
(b) Calculate the statistic F_{Nu}-test based on the u th sample.
(c) Record F_{Nu}-test results $I_{Nu} = 1$ if H_{N0} is rejected at the significance level α and accepted $I_{Nu} = 0$.

- Calculate the ratio of significant tests $\widehat{\pi}(\mu_{Nu}) = \frac{1}{a_N}\sum_{u=1}^{a_N} I_{Nu}$. Figure 2 shows the steps of the MC method.

In order to assess the power under the neutrosophic statistics, alternative hypotheses are considered:
$(\mu_{N1}, \mu_{N2}, \mu_{N3}, \mu_{N4}) = (0,1, 1,1)$, $(\mu_{N1}, \mu_{N2}, \mu_{N3}, \mu_{N4}) = (0,1, 2,2)$,
$(\mu_{N1}, \mu_{N2}, \mu_{N3}, \mu_{N4}) = (1,1, 3,3)$, $(\mu_{N1}, \mu_{N2}, \mu_{N3}, \mu_{N4}) = (0,1, 2,3)$,
$(\mu_{N1}, \mu_{N2}, \mu_{N3}, \mu_{N4}) = (0,2, 3,3)$, $(\mu_{N1}, \mu_{N2}, \mu_{N3}, \mu_{N4}) = (0,1, 3,4)$.

Discussion and comparative study

The objective of this section is to evaluate the results obtained in the examples and the simulation study for the proposed design in the presence of uncertainty. According to the literature on neutrosophic logic, a method based on indeterminate data is more effective and suitable for use in uncertain situations than one based solely on determined values. In this light, we will assess the effectiveness of the F_N-test by examining its measure of indeterminacy, adequacy, information, and flexibility. Additionally, the proposed F_N-test will be compared with the statistic of the existing F-test in terms of empirical type I error and the empirical power of the test. For example, in Table 3, the neutrosophic form of the F_{NL}-test for treatments (Latin letters) is $13.35 + 15.56I_{F_{NL}}$; $I_{F_{NL}} \in [0, 0.142]$. There are two parts to this neutrosophic form: an F-test of classical statistics and an indeterminate part. The neutrosophic form of the neutrosophic F_N-test reduces to the F-test when $I_{F_N} = 0$. In other words, the value 13.35 represents the F-test value under classical statistics for the existing Graeco-Latin square design. As for the second part, $15.56I_{F_{NL}}$, it contains an indeterminate part that has a measure of indeterminacy of 0.142. On the other hand, at α significance level, the $p_N - value$ is $[0.030, 0.025] < 0.05$. In light of this, the neutrosophic null hypothesis is rejected while the neutrosophic alternative hypothesis is accepted. This indicates that there are significant differences between the means of the assumed treatments (Latin letters).

Moreover, Table 6 and Fig. 3 present empirical type I error rates and power of tests under NS, showing results within the indeterminate interval that is expected under uncertain conditions. As can be seen from Fig. 3, the curve of the power of test for the indeterminate part lies above the curve for the determinate part. This emphasizes the importance of the indeterminate part in uncertain environments. In light of the results of the study, it can be concluded that the proposed F_N-test is more informative and flexible than the existing F-test.

Table 6 The simulation results for (Row = 4, Column = 4, Latin letter = 4, Greek letter = 4)

Test	α	Mean empirical type I error	Mean empirical power					
			$\delta_1 = (0,1,1,1)$	$\delta_2 = (0,1,2,2)$	$\delta_3 = (1,1,3,3)$	$\delta_4 = (0,1,2,3)$	$\delta_5 = (0,2,3,3)$	$\delta_6 = (0,1,3,4)$
NGLSD	0.01	[0.0110, 0.0113]	[0.0223, 0.0255]	[0.0709, 0.0804]	[0.1028, 0.1142]	[0.1265, 0.1421]	[0.1593, 0.1814]	[0.2667, 0.3012]
	0.025	[0.0242, 0.0243]	[0.0555, 0.0633]	[0.1570, 0.1775]	[0.2142, 0.2440]	[0.2665, 0.3039]	[0.3149, 0.3599]	[0.4961, 0.5529]
	0.05	[0.0470, 0.0489]	[0.1075, 0.1166]	[0.2744, 0.3053]	[0.3865, 0.4259]	[0.4416, 0.4808]	[0.5161, 0.5623]	[0.7178, 0.7657]
	0.10	[0.1026, 0.1029]	[0.2059, 0.2208]	[0.4580, 0.4946]	[0.5786, 0.6289]	[0.6730, 0.7173]	[0.7331, 0.7813]	[0.8945, 0.9245]

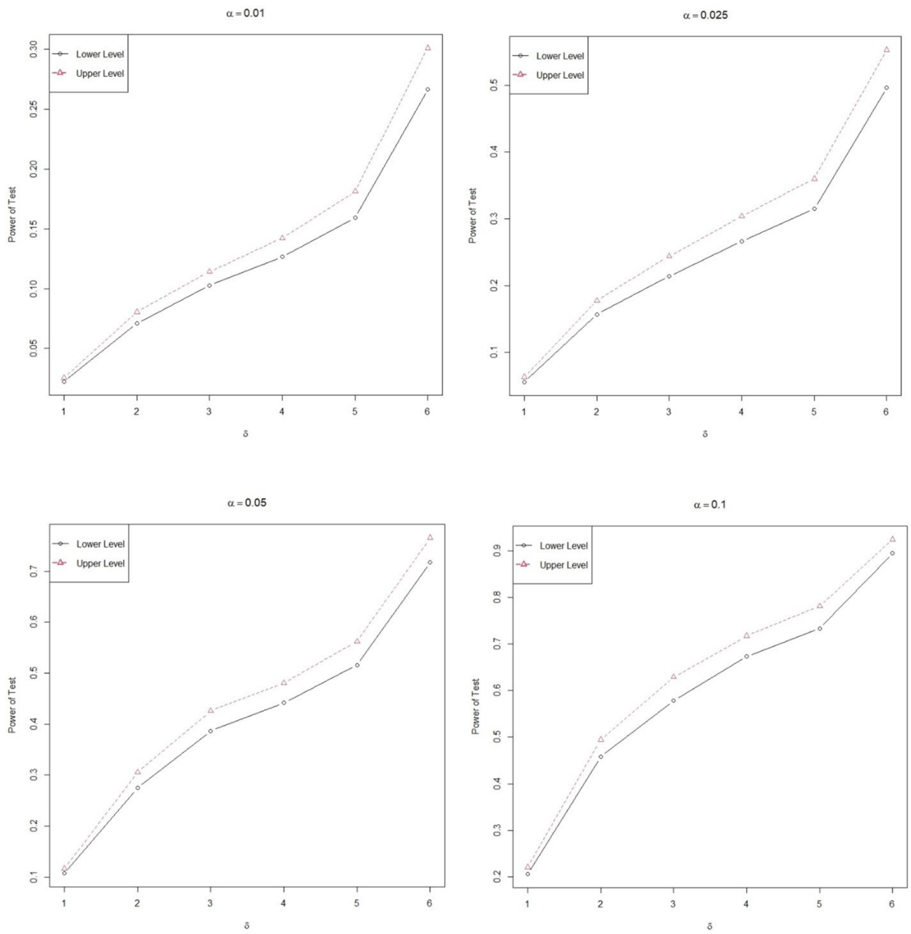

Fig. 3 Power of test for NGLSD

Conclusion

It is the aim of the present paper to propose a Graeco-Latin square design under NS which is suitable for the analysis of indeterminate, uncertain, and imprecise data. A statistical model and a NANOVA approach have been presented for the proposed design. Furthermore, neutrosophic hypotheses were identified as well as a decision rule for the proposed design. Numerical examples and simulation studies were conducted to evaluate the proposed design. According to the results, the proposed F_N-test offers greater flexibility, applicability, and information when compared with the existing F-test in presence of uncertain data. Accordingly, we recommend that researchers use the proposed design rather than the existing design when working in uncertain environments.

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