## RESEARCH



# Simulating imprecise data: sine-cosine and convolution methods with neutrosophic normal distribution

Muhammad Aslam<sup>1\*</sup>

\*Correspondence: aslam\_ravian@hotmail.com

<sup>1</sup> Department of Statistics, Faculty of Science, King Abdulaziz University, 21551 Jeddah, Saudi Arabia

## Abstract

**Objective:** The primary aim of this research paper is to introduce and demonstrate the application of the sine–cosine method and the convolution method for simulating data by utilizing the neutrosophic normal distribution.

**Method:** The methodological framework presented in this paper elaborates on the incorporation of both the sine–cosine method and the convolution method into the realm of neutrosophic statistics. It also introduces algorithms engineered to produce random variables adhering to the neutrosophic normal distribution.

**Results:** Moreover, the study furnishes practical tables that encompass neutrosophic random normal variables generated via the sine–cosine method, as well as tables exhibiting neutrosophic random standard normal variables generated using the convolution method.

**Conclusion:** The analysis undertaken in this study conclusively establishes that the proposed sine–cosine and convolution simulation methods yield outcomes presented in the form of intervals. Furthermore, the study's conclusion emphasizes that the extent of indeterminacy significantly influences the characteristics of the random variates.

**Keywords:** Classical method of simulation, Uncertainty, Neutrosophy, Normal distribution, Simulation

## Introduction

Random numbers hold significant importance across various domains, including experimental design, statistical analysis, cryptography, and diverse disciplines, as highlighted by [26]. In earlier times, random numbers were derived through methods such as coin flipping and other simulation techniques. However, with the advent of high-speed computing, contemporary approaches employ computer-based generators to produce random variations efficiently. These generators are tailored to the specific problems at hand and involve algorithmic designs aimed at generating random variables for a range of applications. Among various simulation methodologies, the sine–cosine simulation method and the convolution simulation method have gained substantial traction, particularly in the generation of standard normal variates. These techniques involve the creation of continuous uniform numbers



© The Author(s) 2023. **Open Access** This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http:// creativecommons.org/licenses/by/4.0/.

sourced from the continuous uniform distribution. Subsequently, these numbers are leveraged to generate standard normal variates. The sine-cosine method was introduced by [8]. Applied [13] the sine-cosine algorithm in solving the issues in machine scheduling problems [11]. used sine-cosine transformation in developing the probability distribution [18]. Discussed the algorithm to generate random variate from the normal distribution [7]. Applied both the sine-cosine and convolution methods in a neural network application [19]. Implemented the sine-cosine algorithms within a wastewater treatment system. More information on sine-cosine and convolution transformation can be seen in [12, 22] and [16]. Various studies on generating random numbers can be seen in [4, 14, 21] and [27].

The neutrosophic statistics (NS) introduced by [25] and applied when the observations in the data are in an interval or have imprecise observations. The NS was found to be more efficient than classical statistics (CS) in terms of flexibility and information [9, 10]. Presented a methodology to deal with neutrosophic numbers [20]. Introduced the t-test for the AR (1) process [2]. Outlined the experimental design within the framework of neutrosophic statistics [17]. Engaged with the probabilistic methodology employing neutrosophic statistics [1]. Investigated a discrete distribution utilizing neutrosophic statistics [5]. Expanded the theory of survey sampling through the application of neutrosophic statistics [6]. conducted the research on the neutrosophic statistical test for sequential contingency. The various applications of NS can be seen in [3, 15] and [23].

The current sine-cosine and convolution simulation techniques within the framework of classical statistics face limitations in simulating standard normal variates in the context of uncertainty. A thorough literature review, conducted to the best of the authors' knowledge, reveals a lack of research on sine-cosine and convolution simulation methods within the framework of neutrosophic statistics. This paper will introduce the routines and algorithms for both simulation methods. Neutrosophic standard normal tables for both the sine-cosine and convolution simulation methods will be presented and analyzed in detail. The significance of this work lies in its pioneering approach to addressing a gap in simulation methods within the realm of uncertainty. By introducing and exploring sine-cosine and convolution simulation methods under the framework of neutrosophic statistics, this research contributes to expanding the toolkit available for handling complex and uncertain data scenarios. As existing methods struggle to simulate standard normal variates accurately in the presence of uncertainty, this work offers a novel solution that has not been explored before in the literature. The presentation of routines, algorithms, and tables for both simulation methods not only fills this research gap but also opens new avenues for statistical analysis in uncertain contexts. Sect. "Preliminaries" provides an introduction to neutrosophic random variables. In Sect. "The Proposed Sine-Cosine Simulation Method", the Sine-Cosine Simulation Method is elaborated, followed by the presentation of the Convolution Method of Simulation in Sect. "The Proposed Convolution Method". Sect. "Simulation Studies" encompasses the simulation studies, while the final section offers concluding remarks.

## Preliminaries

Suppose that  $X_N \epsilon[X_L, X_U]$  is a neutrosophic random variable follows the neutrosophic normal distribution having uncertain in the mean or in standard deviation or in mean and standard deviation. Let  $\mu_N \epsilon[\mu_L, \mu_U]$  be mean and  $\sigma_N \epsilon[\sigma_L, \sigma_U]$  be the standard deviation of the neutrosophic normal distribution. The neutrosophic forms of  $X_N \in [X_L, X_U]$  is expressed as

$$X_N = X_L + X_U I_N; I_N \epsilon [I_L, I_U] \tag{1}$$

The neutrosophic forms of  $\mu_N \epsilon[\mu_L, \mu_U]$  is expressed as

$$\mu_N = \mu_L + \mu_U I_N; I_N \epsilon [I_L, I_U] \tag{2}$$

The neutrosophic forms of  $\sigma_N \epsilon[\sigma_L, \sigma_U]$  is expressed as

$$\sigma_N = \sigma_L + \sigma_U I_N; I_N \in [I_L, I_U] \tag{3}$$

Note that the first values in the above neutrosophic forms denote the values under classical statistics and the second values denote the determinate part and  $I_N$  is the measure of uncertainty. Note that the neutrosophic forms reduce to determinate values when  $I_L = 0$ .

The neutrosophic probability density function (npdf) of the neutrosophic normal distribution is given by

$$f(X_N) = \frac{1}{\sigma_N \sqrt{2\pi}} \exp\left(-\frac{(X - \mu_N)^2}{2\sigma_N^2}\right)$$
(4)

The neutrosophic standard normal distribution is expressed by

$$f(Z_N) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(z)^2}{2}\right)$$
(5)

More details about the neutrosophic normal distribution and its shape can be seen in [24].

#### The proposed sine-cosine simulation method

Suppose that  $U_{1N} \in [U_{1L}, U_{1U}]$  and  $U_{2N} \in [U_{2L}, U_{2U}]$  are the uniform continuous random numbers for the same neutrosophic continuous uniform distribution. Note that  $U_{1N}$  and  $U_{2N}$  are uniform continuous random numbers ranging from 0 to 1. By following [8], two neutrosophic standard normal random variates  $z_{1N}$  and  $z_{2N}$  can be generated as follows

$$z_{1N}\epsilon\left[\left\{\sqrt{-2\ln U_{1L}}\cos(2\pi(U_{2L}))\right\},\left\{\sqrt{-2\ln U_{1U}}\cos(2\pi(U_{2U}))\right\}\right]$$
(6)

$$z_{2N} \epsilon \left[ \left\{ \sqrt{-2 \ln U_{1L}} \sin(2\pi (U_{2L})) \right\}, \left\{ \sqrt{-2 \ln U_{1U}} \sin(2\pi (U_{2U})) \right\} \right]$$
(7)

The joint neutrosophic density  $X_{1N} \in [X_{1L}, X_{1U}]$  and  $X_{2N} \in [X_{2L}, X_{2U}]$  can be given by

$$f(z_{1N}, z_{2N}) = \left[ \left\{ \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(z_{1L})^2}{2}\right), \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(z_{1U})^2}{2}\right) \right\}.$$
$$\left\{ \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(z_{2L})^2}{2}\right), \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(z_{2U})^2}{2}\right) \right\} \right] = f(z_{1N})f(z_{2N})$$
(8)

By following [8], it is concluded that  $z_{1N} \in [z_{1L}, z_{1U}]$  and  $z_{2N} \in [z_{2L}, z_{2U}]$  are independent random variate from the same neutrosophic standard normal distribution in Eq. (5). The process for generating neutrosophic standard normal and neutrosophic normal random variates is elucidated as follows:

**Step-1:** Generate two neutrosophic continuous uniform random numbers  $U_{1N} \in [U_{1L}, U_{1U}]$  and  $U_{2N} \in [U_{2L}, U_{2U}]$  from 0 to 1.

**Step-2:** Generate the neutrosophic standard random variable  $z_{1N} \in [z_{1L}, z_{1U}]$  using  $z_{1N} = \left[\left\{\sqrt{-2\ln U_{1L}}\cos(2\pi(U_{2L}))\right\}, \left\{\sqrt{-2\ln U_{1U}}\cos(2\pi(U_{2U}))\right\}\right].$ 

**Step-3:** Generate the neutrosophic standard random variable  $z_{2N} \in [z_{2L}, z_{2U}]$  using  $z_{2N} = \left[\left\{\sqrt{-2\ln U_{1L}}\sin(2\pi(U_{2L}))\right\}, \left\{\sqrt{-2\ln U_{1U}}\sin(2\pi(U_{2U}))\right\}\right]$ 

**Step-4:** Return  $z_{1N} \epsilon[z_{1L}, z_{1U}]$  and  $z_{2N} \epsilon[z_{2L}, z_{2U}]$ 

**Step-5:**  $x_{1N} = \mu_N + z_{1N} \times \sigma_N$  and  $x_{2N} = \mu_N + z_{2N} \times \sigma_N$ 

**Step-6:** Return  $x_{1N}$  and  $x_{2N}$ 

The operational procedure of the proposed simulation method is also shown in Fig. 1.



Fig. 1 Algorithm of sine-cosine method



Fig. 2 Algorithm of the convolution method

#### The proposed convolution method

This section delves into the convolution method within the framework of neutrosophic statistics, drawing inspiration from [26]. The presented simulation technique operates as follows: utilizing a set (Fig. 2) of twelve neutrosophic continuous uniform random numbers to produce neutrosophic standard normal variates. The routine for implementing the proposed convolution method under the domain of neutrosophic statistics is outlined as follows:

**Step-1:** Generate continuous uniform random numbers  $U_N \in [U_L, U_U]$  from 0 to 1.

**Step-2:** Set neutrosophic standard normal variable  $Z_N \in [Z_L, Z_U]$ .

**Step-3:** For i=1 to 12, calculate the sum of 12 continuous uniform random variates and compute  $Z_{N1} = Z_N + \sum U_N$ ;  $Z_{N1} \in [Z_{L1}, Z_{U1}]$ ,  $U_N \in [U_L, U_U]$ 

**Step-4:** return  $Z_{N1} \in [Z_{L1}, Z_{U1}]$ 

### **Simulation studies**

Within this segment, we will generate random normal variates using the sine-cosine method across a range of specified parameter values. Simultaneously, we will employ the convolution method to generate standard normal variates. Our approach involves adhering to the prescribed algorithms to generate these random standard normal variates.

#### Sine-cosine simulation

Tables 1, 2, 3, 4 exhibit the results obtained using the proposed sine-cosine method, showcasing two random normal variates, denoted as  $x_{1N}$  and  $x_{2N}$ . These

£rs	<i>I<sub>N</sub>=</i> 0		I <sub>N</sub> =0.01		I <sub>N</sub> =0.02		I <sub>N</sub> =0.03		I <sub>N</sub> =0.05		I <sub>N</sub> =0.10	
	NLX	X2N	NLX	NZX	NLX	NZX	NLX	X_2N	NLX	X2N	NLX	X <sub>2</sub> N
<i>—</i>	[9.2,10.8]	[11.6,14.4]	[9.4,11]	[11.2,13.7]	[10.9,13.5]	[8.9,10.5]	[10.2,12.8]	[7.8,8.8]	[10.2,12.4]	[9.5,11.54]	[9.5,11.54]	[10.1,11.77]
2	[10, 12]	[11.3,14]	[10.5,12.8]	[8.2,9.4]	[10.6,13.1]	[8.4,9.7]	[9.6,11.7]	[8.7,10.1]	[8.9,10]	[12.2,15.06]	[12.2,15.06]	[9.1,10.73]
m	[8.2,9.3]	[9.6,11.4]	[10,12.1]	[9.5,11.3]	[10.9,13.5]	[8.6,10.1]	[11.6,14.7]	[8.6,10.4]	[11.2,13.6]	[12.4,15.61]	[12.4,15.61]	[9.2,11.9]
4	[9.9,11.9]	[11.2,13.8]	[9.2,10.8]	[9.3,10.9]	[10.6,12.8]	[10.5,12.7]	[9.1,10.8]	[9.8,11.6]	[10.4,13]	[8.9,10.59]	[8.9,10.59]	[9.3,11.51]
5	[10.3,12.5]	[10.6,12.9]	[12.7,16.1]	[10.7,13.1]	[9.9,11.8]	[11.5,14.3]	[9.9,12.1]	[8.6,10]	[11,13.4]	[10.7,13.05]	[10.7,13.05]	[10.1,12.23]
9	[10.6,12.8]	[6.8,7.2]	[9.1,10.6]	[10.6,12.9]	[10.4,12.5]	[11.8,14.8]	[10,12]	[10.6,12.8]	[9.5,11.4]	[9.5,11.24]	[9.5,11.24]	[9.5,11.7]
7	[9.9,11.9]	[11,13.6]	[11,13.5]	[10.3,12.5]	[10.4,12.5]	[10.6,12.9]	[9.6,11.4]	[10.4,12.5]	[9,10.8]	[9.2,10.57]	[9.2,10.57]	[9.5,11.06]
∞	[9,10.5]	[10.5,12.7]	[9.4,11.2]	[9.7,11.5]	[11.1,13.6]	[11.3,14]	[10.3,12.4]	[9.5,11.4]	[10.9,13.2]	[12.1,15.13]	[12.1,15.13]	[9.9,11.74]
6	[10.4,12.6]	[11.8,14.7]	[11.3,14]	[9,10.6]	[8.9,10.5]	[9.3,10.8]	[10.3,12.3]	[11.4,14.1]	[11,13.5]	[10,12.05]	[10,12.05]	[9.6,10.71]
10	[10.9,13.3]	[10.7,13.1]	[10.6,12.9]	[10.2,12.3]	[10.5,13]	[8.4,9.8]	[9.1,10.5]	[11.3,13.8]	[10.9,13.3]	[10.2,12.36]	[10.2,12.36]	[6,11]
11	[9.3,11]	[10.4,12.6]	[11.3,13.9]	[11,13.6]	[9.5,11.3]	[9.6,11.3]	[9.7,11.8]	[8.8,10.2]	[10,12.2]	[9.4,11.17]	[9.4,11.17]	[9.8,11.59]
12	[9.3,10.9]	[10.3,12.4]	[10,12.1]	[8.1,9.2]	[8.9,10.4]	[10.5,12.6]	[8.7,10.1]	[10.1,11.9]	[9.3,11]	[10.4,12.4]	[10.4,12.4]	[9.8,12.5]
13	[10.5,12.8]	[11.4,14.1]	[11.5,14.2]	[10.2,12.3]	[8.7,10.1]	[10,11.9]	[9,10.7]	[9.3,10.9]	[8.6,9.8]	[11.1,13.38]	[11.1,13.38]	[7.1,8.09]
14	[9.5,11.2]	[9.4,11.1]	[9.1,10.7]	[10,11.9]	[8.4,9.7]	[9.4,11]	[9.8,11.6]	[10.8,13.1]	[9.4,11.2]	[9.6,11.32]	[9.6,11.32]	[9.8,12.41]
15	[10.4,12.5]	[10.9,13.3]	[10.3,12.6]	[9.2,10.8]	[10.2,12.2]	[10.8,13.1]	[9.5,11.1]	[11,13.4]	[8.5,10]	[9.3,10.61]	[9.3,10.61]	[11.6,14.23]
16	[9.5,11.3]	[9.9,11.9]	[9.3,10.9]	[10.8,13.2]	[10.1,12.1]	[11.4,14.1]	[10.3,12.4]	[10.4,12.5]	[8.6,10.1]	[9.3,10.55]	[9.3,10.55]	[10,11.67]
17	[11.5,14.3]	[10.2,12.3]	[9,10.5]	[9.7,11.5]	[10,11.9]	[11.5,14.2]	[12.6,15.9]	[11.2,13.8]	[10.4,12.5]	[10.6,12.83]	[10.6,12.83]	[8.2,8.88]
18	[10.6,12.9]	[10.3,12.4]	[8.6,9.9]	[10.8,13.1]	[11.1,13.7]	[10.7,13.1]	[10,12.3]	[8.8,10.2]	[8.3,9.7]	[9.5,10.84]	[9.5,10.84]	[10.9,13.1]
19	[10.9,13.4]	[10,12]	[9.3,11]	[9.6,11.4]	[9.2,10.7]	[10.6,12.8]	[9.7,11.4]	[11,13.5]	[9.8,11.6]	[10.7,12.99]	[10.7,12.99]	[9.9,11.55]
20	[8.1,9.2]	[7.8,8.6]	[11,13.5]	[10.1,12.2]	[9.2,10.9]	[9,10.5]	[10.3,12.4]	[10.3,12.4]	[10.7,13.7]	[8.1,9.57]	[8.1,9.57]	[10.8,13.41]
21	[9.8,11.7]	[11.3,14]	[9.3,10.9]	[11.2,13.7]	[10.5,12.7]	[9.6,11.4]	[10.1,12.3]	[8.9,10.5]	[10.4,12.5]	[10.7,12.93]	[10.7,12.93]	[10.6,13.46]
22	[9.5,11.2]	[10.7,13]	[11.9,14.9]	[11,13.5]	[11.1,13.7]	[10.5,12.7]	[9.8,11.6]	[11,13.4]	[9.5,11.2]	[10.6,12.66]	[10.6,12.66]	[10.3,12.3]
23	[9.2,10.8]	[10.5,12.8]	[9.7,11.6]	[11,13.5]	[8.3,9.7]	[8,8.9]	[7.4,8.2]	[9.4,10.7]	[9.4,11.1]	[10.5,12.52]	[10.5,12.52]	[10.6,13.06]
24	[7.9,8.9]	[10.5,12.7]	[8.2,9.3]	[12.1,15.2]	[1 0.2, 1 2.2]	[11.5,14.3]	[9.6,11.4]	[9.9,11.8]	[8.9,10.4]	[10.1,11.86]	[10.1,11.86]	[11.3,13.71]
25	[10,12]	[11.2,13.9]	[9.8,11.7]	[10.9,13.3]	[9.6,11.5]	[8.8,10.2]	[9,10.6]	[9.9,11.7]	[9.7,11.4]	[11,13.41]	[11,13.41]	[10.4,12.32]

**Table 1** Random normal variate when  $\mu = [10, 12]$  and  $\sigma = [1, 1.5]$ 

#JS	I <sub>N</sub> =0		I <sub>N</sub> =0.01		I <sub>N</sub> =0.02		I <sub>N</sub> =0.03		I <sub>N</sub> =0.05		<i>I<sub>N</sub></i> =0.10	
	X1N	NZN	NLX	X2N	NLX	X2N	NLX	XZN	X1N	X2N	NLX	X2N
	[9.2,10.7]	[15.8,21.6]	[7.8,8.3]	[12.7,16.3]	[13,17]	[10.5,12.8]	[12.2,15.4]	[11.7,14.7]	[13.8,18.1]	[12.8,16.85]	[12.8,16.85]	[9.8,10.28]
2	[7.9,8.5]	[9.9,11.9]	[7.3,7.7]	[6.8,6.5]	[13.1,17.1]	[11.2,13.9]	[4.9,4.1]	[7.1,6.3]	[8,8.7]	[11.1,13.27]	[11.1,13.27]	[8.4,10.56]
ε	[9.4,11]	[7.7,8.2]	[9.1,10.9]	[5.3,4.1]	[13.4,17.6]	[10.1,12.2]	[5.9,4.8]	[1 3.8, 1 7.9]	[9.8,12.8]	[6.9,7.07]	[6.9,7.07]	[9,11.65]
4	[5.7,4.8]	[10.4,12.6]	[11.3,14]	[16.4,22.6]	[5.9,5.1]	[10.9,13.1]	[14.5,19.3]	[12.4,16]	[13.8,18.1]	[10.9,13.46]	[10.9,13.46]	[10.5,12.38]
5	[8.3,9.2]	[6.8,6.7]	[6.8,6.7]	[9.9,11.6]	[11.6,14.5]	[13.6,18]	[8.3,9]	[12.2,15.3]	[11.3,13.8]	[12.8,16.64]	[12.8,16.64]	[7.4,9.19]
9	[11.7,14.8]	[12,15.3]	[12.5,16.5]	[6.3,6]	[7.3,7.8]	[8.6,9.4]	[7.4,8]	[9.7,11.1]	[11,16.4]	[3.1,1.38]	[3.1,1.38]	[10,17.86]
7	[9.2,10.7]	[13.1,17.1]	[9.7,11.7]	[7.4,7.7]	[12.3,15.6]	[14,18.7]	[15,21.1]	[6.4,7.5]	[10.3,12.1]	[13.1,16.95]	[13.1,16.95]	[13,16.57]
œ	[6.3,5.8]	[5.5,4.5]	[12.6,16.3]	[12.3,15.8]	[5.6,4.3]	[15,20]	[4.3,3.2]	[7.1,6.2]	[8.9,12.7]	[3.6,1.24]	[3.6,1.24]	[10.3,14.35]
6	[11.9,15.2]	[13.7,18.1]	[11.6,15.1]	[5.9,5.3]	[6.1,5.7]	[8.6,9.2]	[10.5,13.2]	[8.2,9.4]	[7.2,10.2]	[2.3,-1.47]	[2.3,-1.47]	[13.4,17.1]
10	[10.9,13.4]	[3.1,0.5]	[6.8,6.7]	[10.5,12.6]	[11.1,13.7]	[12.1,15.4]	[5.6,4.6]	[11.7,14.3]	[5.2,3.2]	[14.8,18.9]	[14.8,18.9]	[9.9,15.38]
11	[7,7]	[8.5,9.5]	[8.6,9.8]	[11,13.6]	[8.6,9.9]	[9.6,11.3]	[9,10.3]	[11.5,14.2]	[4.7,3.3]	[10.5,11.52]	[10.5,11.52]	[12.5,20.03]
12	[10.9,13.6]	[13.4,17.7]	[11.7,14.8]	[10.9,13.4]	[4.3,3.1]	[6.2,5]	[8.1,9.1]	[10.2,12.1]	[10,11.7]	[12,14.95]	[12,14.95]	[10.6,15.41]
13	[9.7,11.6]	[15.6,21.4]	[9.3,10.9]	[11.5,14.4]	[8.2,9.3]	[9.6,11.3]	[8.8,10.6]	[7.6,7.9]	[8.8,9.7]	[12.7,16]	[12.7,16]	[8.6,11.23]
14	[5,3.7]	[12.9,16.9]	[10.5,12.7]	[12.7,16.5]	[8.8,10.5]	[6.8,6.6]	[11.1,13.7]	[12.1,15.4]	[13,16.6]	[14.5,19.66]	[14.5,19.66]	[12.4,15.2]
15	[5.9,5.2]	[14.6,19.6]	[16.9,23.5]	[11.2,14]	[14,18.4]	[15,20.4]	[12.2,15.5]	[13,17.1]	[10.4,14.2]	[6,5.94]	[6,5.94]	[8,12.08]
16	[5.8,4.9]	[5.1,3.8]	[5.5,4.2]	[15.6,21.2]	[11,13.5]	[11.6,14.5]	[17,23.6]	[10.8,13.3]	[2.7,-0.7]	[13.4,15.87]	[13.4,15.87]	[5.5,5.23]
17	[10.8,13.3]	[8.9,10.1]	[7.8,8.2]	[16.5,22.8]	[5.8,4.9]	[12.1,15.2]	[7.4,7.6]	[11.5,14.1]	[10.8,13.7]	[8.4,10.17]	[8.4,10.17]	[10,15.36]
18	[12.4,16.1]	[12,15.4]	[10.5,12.8]	[11.4,14.3]	[5,3.9]	[9.1,9.9]	[8,9.1]	[8.7,9.6]	[10.4,12.4]	[11.5,14.03]	[11.5,14.03]	[7,7.64]
19	[14.2,18.9]	[12.5,16.2]	[12.4,15.9]	[10.9,13.5]	[12,15.6]	[8.1,9.3]	[10.4,12.5]	[11.4,14]	[10.6,12.3]	[15.4,20.9]	[15.4,20.9]	[13.5,19.18]
20	[9.5,11.1]	[5.7,4.9]	[8.4,9.3]	[11.3,14.1]	[8.1,9]	[9.8,11.5]	[7.7,8]	[12.2,15.2]	[7.1,7.2]	[10.7,12.39]	[10.7,12.39]	[3.8,3.34]
21	[17.5,24.5]	[11.6,14.7]	[8.6,9.8]	[9.5,11.1]	[9.1,10.6]	[11.1,13.7]	[5.5,5.2]	[6.8,5.9]	[13.3,17.3]	[11.1,13.93]	[11.1,13.93]	[10.5,12.32]
22	[12,15.3]	[14.8,20]	[16.3,22.4]	[12,15.4]	[15.4,20.9]	[14.7,19.9]	[14.6,19.4]	[13.2,17.4]	[6.3,5.9]	[10.7,12.21]	[10.7,12.21]	[7.3,7.48]
23	[11.6,14.6]	[12.1,15.5]	[9.8,11.5]	[14.3,19.2]	[11.4,15.3]	[4.3,2.9]	[12.5,16]	[13.1,17.2]	[5.8,4.5]	[12.7,15.47]	[12.7,15.47]	[12.4,16.76]
24	[9.4,11.1]	[8.5,9.4]	[11,13.8]	[6.8,6.9]	[9.3,10.8]	[11.6,14.5]	[7.3,7.3]	[12.3,15.4]	[15,20.2]	[10.3,12.58]	[10.3,12.58]	[7.6,8.29]
25	[12.2,15.6]	[10.8,13.3]	[8.7,10.1]	[6.9,6.8]	[6,5]	[16,21.7]	[9.2,10.6]	[11.6,14.3]	[7.2,8.7]	[6.3,5.17]	[6.3,5.17]	[6.5,7.06]

[3, 5]
, 12] and $\sigma = [$
$\mu = [10]$
variate whe
Random normal
Table 2

Sr#	<i>I<sub>N</sub>=</i> 0		I <sub>N</sub> =0.01		I <sub>N</sub> =0.02		I <sub>N</sub> =0.03		I <sub>N</sub> =0.05		I <sub>N</sub> =0.10	
	NIX	X2N	NLX	X2N	NLX	NZX	NLX	X2N	N1X	X2N	NLX	X <sub>2N</sub>
-	[19.6,21.5]	[20.8,23.2]	[20.4,22.6]	[21.6,24.4]	[21.2,23.8]	[21.7,24.6]	[18.6,19.8]	[20.9,23.1]	[18.9,20.8]	[18.6,19.66]	[18.6,19.66]	[21.5,23.64]
2	[21.6,24.4]	[18.3,19.5]	[20.4,22.6]	[19.7,21.6]	[21.9,24.8]	[20.2,22.3]	[20.9,23.2]	[20.9,23.4]	[18.9,20.2]	[20.9,23.11]	[20.9,23.11]	[21,23.34]
m	[21.3,24]	[22,24.9]	[19.3,20.9]	[20.3,22.5]	[19.1,20.8]	[18.8,20.1]	[20.2,22.2]	[22,25]	[19.8,21.7]	[20.6,22.71]	[20.6,22.71]	[20.4,22.19]
4	[20.3,22.5]	[20.7,23]	[20.6,22.8]	[20.4,22.6]	[18.4,19.7]	[19.9,21.8]	[18,19.1]	[19.7,21.2]	[20.8,23.1]	[20.1,22.21]	[20.1,22.21]	[19.6,21.24]
5	[19.2,20.8]	[21.1,23.7]	[20.3,22.4]	[20.8,23.2]	[19.8,21.9]	[18.3,19.5]	[19.8,22]	[18.8,20.2]	[19.2,20.7]	[20.9,23.18]	[20.9,23.18]	[20.3,21.95]
9	[20.1,22.2]	[20.9,23.4]	[19.5,21.2]	[21,23.5]	[17.6,18.4]	[20.7,22.9]	[18.8,20.2]	[19.9,21.7]	[19.6,21.4]	[20.5,22.65]	[20.5,22.65]	[20.7,23.26]
7	[20,22]	[21.8,24.8]	[19.5,21.4]	[18.1,19.1]	[19.3,21.1]	[19.2,20.7]	[19.6,21.5]	[19.7,21.6]	[20.1,22.5]	[19,20.67]	[19,20.67]	[21.2,24.25]
00	[18.1,19.2]	[20.4,22.6]	[20.1,22.2]	[20.6,22.8]	[20.8,23.3]	[19,20.6]	[21.3,24]	[20.3,22.5]	[20.8,23.1]	[21.2,23.74]	[21.2,23.74]	[20.8,22.86]
6	[18.3,19.4]	[19.5,21.2]	[18,19]	[19.2,20.7]	[18.4,19.6]	[20.8,23]	[20.6,22.9]	[20.6,22.9]	[20,21.9]	[21.1,23.55]	[21.1,23.55]	[18.8,20.49]
10	[19.6,21.4]	[18.9,20.3]	[20.7,23]	[20.3,22.5]	[18.6,19.9]	[20.5,22.6]	[20.6,22.9]	[21.7,24.6]	[20.9,23.5]	[19.2,21.29]	[19.2,21.29]	[19,21.38]
11	[19.8,21.8]	[20.7,23]	[20.3,22.4]	[20.6,22.9]	[20.2,22.4]	[19.5,21.3]	[19.8,22.1]	[18.4,19.7]	[20.9,23.9]	[18.3,19.87]	[18.3,19.87]	[20,21.98]
12	[20.7,23]	[19.2,20.8]	[19.7,21.5]	[22.4,25.6]	[19.9,21.8]	[20.5,22.7]	[20.5,22.7]	[20.2,22.2]	[20.3,22.8]	[18.9,20.52]	[18.9,20.52]	[20.3,22.87]
13	[20.5,22.7]	[19.1,20.6]	[20.7,23.2]	[18.9,20.5]	[19.4,21.2]	[18.9,20.3]	[21.2,23.7]	[22.5,25.8]	[19.7,21.5]	[20.5,22.54]	[20.5,22.54]	[18.4,20.06]
14	[20.9,23.4]	[20.8,23.2]	[20,22.1]	[18.7,20]	[20.7,23]	[20.1,22.1]	[19.1,20.5]	[20.8,23.1]	[20.3,22.6]	[19.4,21.37]	[19.4,21.37]	[19.3,22.54]
15	[18.4,19.6]	[19.6,21.4]	[20.4,22.7]	[18.7,20]	[21.7,24.8]	[18.6,20.2]	[19.7,21.9]	[18.8,20.2]	[21,23.4]	[21.1,23.66]	[21.1,23.66]	[18.9,20.86]
16	[19.1,20.7]	[19.4,21.1]	[20.5,22.8]	[19.1,20.8]	[21.2,23.9]	[20.2,22.3]	[18.1,19.2]	[19.8,21.5]	[18.4,19.7]	[19.8,21.3]	[19.8,21.3]	[21.4,24.01]
17	[19.8,21.7]	[20.7,23]	[19.9,21.9]	[20.5,22.7]	[18.3, 19.6]	[19.3,20.8]	[21.5,24.1]	[21.6,24.4]	[20.3,22.3]	[20.9,23.3]	[20.9,23.3]	[17.4,17.72]
18	[21.2,23.8]	[20.5,22.7]	[21,23.4]	[21.5,24.3]	[20,22]	[20.7,23]	[22.9,26.4]	[20.6,23]	[20.3,23.2]	[17.9,19.08]	[17.9,19.08]	[19.3,20.33]
19	[19.6,21.4]	[19.8,21.6]	[20,22.1]	[18.5,19.8]	[20.3,22.5]	[20.4,22.5]	[18.4,19.7]	[19.7,21.3]	[19.4,21.3]	[19.7,21.51]	[19.7,21.51]	[21.8,24.55]
20	[20.5,22.7]	[20.1,22.2]	[20.6,22.9]	[20.3,22.5]	[20.1,22.1]	[20.6,22.8]	[18.3,19.9]	[17.5,18]	[18,19.1]	[20.3,22.07]	[20.3,22.07]	[20.9,24.49]
21	[18.7,20]	[19.6,21.4]	[19.8,21.8]	[19.1,20.7]	[19.2,20.7]	[21.8,24.6]	[21.7,24.5]	[21.4,24.1]	[18.7,20.4]	[19.2,20.5]	[19.2,20.5]	[20.5,22.57]
22	[20.2,22.3]	[19,20.5]	[20,22]	[20.9,23.4]	[20.3,22.7]	[18.7,20.1]	[18.8,20.2]	[20.6,22.8]	[19.2,20.4]	[22.5,25.57]	[22.5,25.57]	[20.1,21.85]
23	[20.9,23.4]	[20.4,22.6]	[20.1,22.1]	[21.2,23.8]	[19.1,20.7]	[20.1,22]	[19.4,21.6]	[17.6,18.4]	[21.4,24.1]	[21.3,24.05]	[21.3,24.05]	[20.8,24.23]
24	[18.3,19.5]	[19.9,21.8]	[21.9,24.9]	[18.3, 19.6]	[22.4,25.6]	[20,22]	[19.8,21.7]	[20.5,22.6]	[21.2,23.8]	[20.7,23.12]	[20.7,23.12]	[19.9,22.55]
25	[21.5,24.2]	[20.1,22.1]	[20.4,22.6]	[19.4,21.1]	[19.2,20.9]	[20.2,22.2]	[18.4,19.5]	[20.5,22.5]	[20.5,23]	[19.1,20.98]	[19.1,20.98]	[21,23.38]

**Table 3** Random normal variate when  $\mu = [20, 22]$  and  $\sigma = [1, 1.5]$ 

Sr#	I <sub>N</sub> =0		I <sub>N</sub> =0.01		I <sub>N</sub> =0.02		I <sub>N</sub> =0.03		I <sub>N</sub> =0.05		<i>I<sub>N</sub></i> =0.10	
	NLX	X2N	NLX	NZX	NLX	NZX	NLX	X_2N	NLX	X2N	NLX	X2N
-	[22.5,26.1]	[23.5,27.9]	[21.1,23.8]	[24.5,29.5]	[18.4,19.7]	[17.2,17.2]	[14.8,14.1]	[16.7,23]	[17.7,18.6]	[16.9,17.86]	[16.9,17.86]	[19.5,19.77]
2	[20.7,23.2]	[12.9,10.2]	[14.8,13.7]	[15.9,14.9]	[18.4,19.2]	[22,25.2]	[23.1,27]	[21.8,28.1]	[15.2,15.5]	[18.9,19.81]	[18.9,19.81]	[22.3,24.89]
e	[18.5,19.4]	[21.7,24.9]	[16.7,16.7]	[17.3,17.4]	[19.7,21.5]	[22.2,25.6]	[22.3,25.7]	[21.3,25.1]	[23.1,27.1]	[25,32.19]	[25,32.19]	[16.2,21.06]
4	[17.9,18.5]	[22.8,26.7]	[18,18.8]	[20,21.9]	[18,19]	[17.8,18.1]	[22.7,27.5]	[16,23.4]	[14.5,13.1]	[18.8,19.29]	[18.8,19.29]	[23.1,26.32]
2	[16.7,16.5]	[23.1,27.2]	[18.2,19.2]	[17.9,18.4]	[21.5,24.3]	[26.8,33.4]	[17.8,18.7]	[19.4,27.1]	[25.8,31.9]	[13.9,12.16]	[13.9,12.16]	[20.9,20.47]
9	[17.7,18.2]	[17,17]	[17,17]	[21.6,24.5]	[20.5,23.1]	[17.9,18.7]	[18.5,19.4]	[22.9,18.5]	[17.5,17.3]	[20.7,24.79]	[20.7,24.79]	[17.3,18.98]
7	[23.9,28.5]	[22.2,25.6]	[23.7,28.1]	[21.3,24.1]	[17.6,17.8]	[22.7,26.2]	[21,23.4]	[22.1,12.3]	[17.2,15.5]	[23.1,26.05]	[23.1,26.05]	[25.7,31.82]
00	[18.4,19.3]	[20.6,23]	[21.1,23.7]	[25.5,31.2]	[16.3,15.5]	[26.9,33.2]	[17.4,17.3]	[23.7,25]	[17.3,18.3]	[19.9,20.66]	[19.9,20.66]	[24.9,29.7]
6	[23,26.9]	[22.6,26.3]	[22.4,25.9]	[20.3,22.5]	[24.3,29.1]	[23.9,28.6]	[24,28.5]	[20.3,15.7]	[16.3,14.6]	[22.5,25.58]	[22.5,25.58]	[20.8,23.28]
10	[16.3,15.8]	[19.3,20.8]	[13.8,11.8]	[19.3,20.5]	[19.6,21.2]	[22.7,26.4]	[22.9,26.6]	[23.9,24.6]	[18.4,20.2]	[22.9,26.31]	[22.9,26.31]	[21.6,24.68]
11	[21.4,24.3]	[12.8,10]	[20.4,22.8]	[18.5,19.7]	[22.2,25.6]	[20,22]	[25.7,32.4]	[15.9,23.5]	[17.7,18.7]	[24.2,28.46]	[24.2,28.46]	[22.6,26.53]
12	[24.3,29.1]	[21.6,24.7]	[23.2,27.4]	[20.5,22.8]	[23.7,28.1]	[20.3,22.6]	[17.5,17.7]	[21.6,32.4]	[15.9,17.9]	[24,30.16]	[24,30.16]	[16.9,21.15]
13	[15.7,14.9]	[26.4,32.7]	[21.4,24.7]	[15.4,14.4]	[19.8,21.6]	[21.4,24.1]	[16.8,17.1]	[18.6,20.1]	[20.7,22.7]	[22.1,24.93]	[22.1,24.93]	[21.2,23.86]
14	[20.9,23.5]	[17.6,18]	[21.4,24.7]	[16.6,16.5]	[15.7,14.7]	[22.1,25.2]	[12,9.7]	[14.6,22.4]	[25.8,31.5]	[21.1,22.98]	[21.1,22.98]	[19,21.82]
15	[21,23.7]	[21.7,24.9]	[22.3,26.3]	[15.3,14.5]	[18,18.9]	[18.7,19.7]	[21,24.2]	[17.7,13.3]	[19.8,20.2]	[24,32.19]	[24,32.19]	[14.6,17.72]
16	[21.2,24]	[16.8,16.7]	[20.1,22.1]	[22.7,26.4]	[15.5,14.7]	[19.8,21.3]	[20.2,22.9]	[17.6,20.4]	[22.8,26.3]	[19.2,21.9]	[19.2,21.9]	[18.5,20.13]
17	[16.8,16.7]	[18.2,18.9]	[21.2,24.3]	[17.2,17.5]	[16,15.5]	[19,19.9]	[19,20.8]	[18.5,30.1]	[20.2,22.3]	[18.6,21.32]	[18.6,21.32]	[19.8,21.65]
18	[23,26.9]	[17.7,18.2]	[19.1,20.8]	[17,17]	[19.8,21.3]	[26.6,32.9]	[20.5,22.6]	[22.3,25.8]	[21.3,24.1]	[22.4,25.41]	[22.4,25.41]	[20.3,22.42]
19	[24.7,29.8]	[21.6,24.7]	[16.3,15.9]	[18.4,19.1]	[19.5,21.6]	[17.4,17.8]	[21.3,25.3]	[15.3,26]	[18.3,20.4]	[17.1,20.37]	[17.1,20.37]	[15.8,14]
20	[18.1,18.8]	[16.7,16.6]	[17.7,18.3]	[19.9,21.6]	[22.4,25.8]	[22.6,26.4]	[20.3,22.2]	[23.2,24.2]	[15.5,15]	[18.9,21.23]	[18.9,21.23]	[20.8,22.33]
21	[23.4,27.7]	[23.6,27.9]	[22.4,25.9]	[21.3,24.2]	[23.8,28.2]	[21.8,25]	[21.1,23.4]	[24.7,25.7]	[23.7,28.2]	[20.5,23.2]	[20.5,23.2]	[18.5,20.9]
22	[12.9,10.2]	[19.8,21.7]	[23.6,27.9]	[20.4,22.7]	[17.1,17.1]	[22,25.1]	[19.1,21]	[18,25.5]	[20.8,23.3]	[20.1,21.69]	[20.1,21.69]	[22.3,25.08]
23	[19.8,21.7]	[22,25.3]	[10.4,6.2]	[17.8,17.8]	[21.3,24]	[22.9,26.9]	[22.1,25.2]	[20.9,13.8]	[25.4,30]	[21.7,25.87]	[21.7,25.87]	[17.8,20.47]
24	[16.6,16.3]	[16.6,16.3]	[22.5,26.1]	[21.9,25.1]	[12.9,10.1]	[22.6,25.7]	[17,16.9]	[21.4,18.2]	[26,31.5]	[18.1,18.65]	[18.1,18.65]	[21.9,24.06]
25	[20.4,22.7]	[23,27]	[17.6,18.1]	[17.6,17.8]	[18.7,19.7]	[22.4,25.8]	[14.4,13.2]	[17.3,18]	[17.9,17.9]	[21.7,27.83]	[21.7,27.83]	[15.6,17.09]

**Table 4** Random normal variate when  $\mu = [20, 22]$  and  $\sigma = [3, 5]$ 

tables provide insights across a range of specified parameter values, encompassing  $\mu_N \epsilon[\mu_L, \mu_U]$  and  $\sigma_N \epsilon[\sigma_L, \sigma_U]$ . The investigation is extended to account for varying levels of indeterminacy, represented as  $I_{U}$ , which influences the generation of random normal variates. In particular, the values of  $I_U$  considered encompass 0, 0.01, 0.02, 0.05, and 0.10, generating pairs of random normal variates. To elaborate, Table 1 is tailored for  $\mu_N \epsilon$  [10, 12] and  $\sigma_N \epsilon$  [1, 1.5], whereas Table 2 corresponds to the parameter ranges  $\mu_N \in [10, 12]$  and  $\sigma_N \in [3, 5]$ . Similarly, Table 3 and Table 4 correspond to distinct parameter combinations, namely  $\mu_N \epsilon$  [20, 22] and  $\sigma_N \epsilon$  [1, 1.5], and  $\mu_N \epsilon$  [20, 22] and  $\sigma_N \epsilon$  [3, 5], respectively. Upon scrutiny of Tables 1, 2, 3, 4, it becomes evident that no discernible trend exists within the neutrosophic random normal variates as the values of  $I_U$  fluctuate between 0 and 0.10. Additionally, the outcomes underscore that the proposed sine-cosine simulation method yields random normal variates represented as intervals, rather than determinate values. For instance, consider the case where  $I_{II}$ = 0.01, as seen in Table 1. Here, the random normal variate spans from 9.4 to 11 in the cosine simulation approach and from 11.2 to 13.7 using the sine method. It is important to note that the values in each interval represent the random normal variate for various degree of indeterminacy.

Sr#	<i>I<sub>N</sub></i> =0	<i>I<sub>N</sub></i> =0.01	I <sub>N</sub> =0.02	I <sub>N</sub> =0.03	I <sub>N</sub> =0.05	<i>I<sub>N</sub></i> =0.10
1	[-1.34,-1.34]	[-0.86,—0.81]	[-0.93,-0.83]	[1.91,2.15]	[-1.04,-0.79]	[-0.31,0.26]
2	[0.65,0.65]	[-0.98,-0.93]	[-0.36,-0.24]	[1.02,1.23]	[0.24,0.55]	[1.1,1.81]
3	[-0.63,-0.63]	[-0.3,-0.24]	[-1.64,-1.56]	[1.49,1.72]	[1.04,1.39]	[-1.16,-0.68]
4	[0.18,0.18]	[-0.07,-0.01]	[-2.26,-2.18]	[1.04,1.25]	[-0.34,-0.06]	[-1.07,-0.58]
5	[1.18,1.18]	[0.56,0.63]	[-0.06,0.05]	[1.08,1.29]	[-2.28,-2.1]	[-0.43,0.12]
6	[-0.44,-0.44]	[-1.16,-1.11]	[0.81,0.95]	[-0.39,-0.22]	[-0.57,-0.29]	[-0.09,0.5]
7	[0.62,0.62]	[-0.7,-0.64]	[-0.48,-0.37]	[-0.05,0.13]	[0.15,0.46]	[0.4,1.04]
8	[-0.01,-0.01]	[-0.23,-0.18]	[-1.14,-1.04]	[-0.5,-0.34]	[-1.21,-0.97]	[-0.41,0.15]
9	[-0.8,-0.8]	[0.33,0.39]	[0.51,0.64]	[-0.69,-0.53]	[0.92,1.27]	[-0.82,-0.3]
10	[1.29,1.29]	[-0.74,-0.69]	[-0.05,0.07]	[-0.13,0.05]	[-0.73,-0.47]	[-0.2,0.38]
11	[-0.58,-0.58]	[0.81,0.88]	[3.03,3.21]	[-2.44,-2.33]	[-0.36,-0.08]	[-1.64,-1.2]
12	[0.86,0.86]	[-0.5,-0.44]	[0.14,0.26]	[-2.1,-1.98]	[-0.14,0.15]	[1.63,2.39]
13	[0.48,0.48]	[-0.34,-0.28]	[0.36,0.49]	[0.37,0.56]	[1.07,1.43]	[0.39,1.03]
14	[0.42,0.42]	[1.67,1.74]	[1.38,1.53]	[0.22,0.41]	[-0.85,-0.59]	[0.3,0.93]
15	[0.08,0.08]	[0.26,0.32]	[0.66,0.79]	[-0.42,-0.25]	[-0.26,0.03]	[-0.34,0.23]
16	[-1.48,-1.48]	[-2.13,-2.09]	[0.92,1.06]	[0.54,0.73]	[-1.74,-1.53]	[0.18,0.8]
17	[0.15,0.15]	[0.82,0.89]	[-1.31,-1.22]	[1.42,1.64]	[-0.37,-0.09]	[-1.57,-1.13]
18	[0.33,0.33]	[-0.18,-0.13]	[-0.58,-0.47]	[1.5,1.72]	[0.51,0.84]	[0.93,1.63]
19	[0.02,0.02]	[0.89,0.96]	[0.63,0.76]	[-0.99,-0.84]	[1,1.35]	[-1.94,-1.53]
20	[-0.82,-0.82]	[0.35,0.42]	[-0.06,0.06]	[-1.45,-1.31]	[0.28,0.59]	[-0.56,-0.02]
21	[0.87,0.87]	[-1,-0.95]	[0.3,0.43]	[-0.43,-0.26]	[0.03,0.33]	[0.72,1.39]
22	[0.92,0.92]	[-1.18,-1.14]	[0.99,1.13]	[-0.8,-0.65]	[0.1,0.4]	[-1.23,-0.75]
23	[-0.65,-0.65]	[0.77,0.83]	[1.05,1.19]	[-0.37,-0.2]	[-0.61,-0.34]	[-0.76,-0.23]
24	[-2.31,-2.31]	[-0.82,-0.77]	[0.96,1.1]	[0.61,0.8]	[1.01,1.36]	[-0.92,-0.41]
25	[0.75,0.75]	[-0.59,-0.54]	[-0.4,-0.29]	[-0.44,-0.27]	[-0.24,0.05]	[-0.02,0.58]

 Table 5
 Standard normal variate from convolution method

#### **Convolution Simulation**

The results of the convolution method yield random standard normal variates for varying levels of indeterminacy, with  $I_{U}$  values of 0, 0.01, 0.02, 0.05, and 0.10. The convolution method's algorithm is utilized to generate these random standard normal variates, as per the prescribed approach. The resulting values are compiled in Table 5. Analyzing Table 5 reveals that the convolution method produces standard normal variates represented as intervals characterized by indeterminacy. These intervals, rather than offering precise numerical values, capture the uncertainty inherent in the process. The proposed simulation approach follows this trend, providing random standard normal variates presented as intervals. For instance, consider the case where  $I_U = 0.01$ ; the resulting random standard normal variate spans from -0.86 to -0.81.

### **Comparative study**

This section aims to contrast the techniques for generating random variates based on neutrosophic statistics and classical statistics. In the proposed simulation methods, a concept of indeterminacy, denoted as  $I_{U}$ , is introduced within the parameters  $\mu_N$  and  $\sigma_N$ . The neutrosophic expressions for  $\mu_N$  and  $\sigma_N$  take the form  $\mu_N = \mu_L + \mu_U I_N$ ;  $I_N \in [I_L, I_U]$  and  $\sigma_N = \sigma_L + \sigma_U I_N$ ;  $I_N \in [I_L, I_U]$ . It is worth noting that these neutrosophic expressions simplify to the mean and variance of the normal distribution when  $I_L$  equals zero. For instance, in the scenario where  $\mu_N = [10, 12]$  and  $\sigma_N = [1,1.5]$ , the corresponding neutrosophic forms of mean and standard deviation will be  $\mu_N = 10 + 12I_N$  and  $\sigma_N = 1 + 1.5I_N$ , respectively. Setting  $I_L$  to zero in these expressions results in the classical statistics' mean and standard deviation values of 10 and 1, respectively. Tables 1, 2, 3, 4, 5 showcase results for various values of  $I_U$ . The initial column in these tables represents the random variable outcomes under classical statistics. To illustrate, considering the sine-cosine simulation method within the realm of classical statistics, Table 1 demonstrates that the random normal variate  $x_{1N}$ falls within the range of 9.2 to 10.8. However, when  $I_{U}$  equals 0.02, this range shifts to 10.9 to 13.5. From this comparative analysis, it becomes evident that the degree of indeterminacy significantly impacts the random variate. Consequently, in situations involving uncertainty, relying on simulation methods rooted in classical statistics might lead decision-makers astray.

#### **Concluding remarks**

The paper introduced the sine-cosine method for generating random normal variates and also presented the convolution method for generating random standard normal variates. Algorithms for both methods were provided as well. Various tables were presented, showcasing the impact of indeterminacy on mean and standard deviation. The outcomes of the comparative study underscored that the degree of indeterminacy influences random normal variates. The proposed methods and algorithms offer practical solutions for simulation in scenarios where precise data recording might be challenging. These simulation methods find applicability in diverse fields including reliability analysis, engineering and manufacturing, healthcare, and environmental modeling. As a potential direction for future research, there is room for an extensive investigation into adapting the proposed sine-cosine simulation method and convolution method for neutrosophic multivariate normal distributions. This could enhance the applicability and scope of the method for more complex and multidimensional data scenarios.

#### Acknowledgements

The author is deeply thankful to the editor and reviewers for their valuable suggestions to improve the quality and presentation of the paper.

**Author contributions** M.A wrote the paper.

Fundina

None.

**Availability of data and materials** The data is given in the paper.

#### Declarations

**Ethics approval and consent to participate** Not applicable.

**Consent for publication** Not applicable.

#### **Competing interests**

The authors declare no conflict of interest.

Received: 9 June 2023 Accepted: 6 September 2023 Published online: 16 September 2023

#### References

- 1. Ahsan—ul-Haq M, Zafar J. A new one-parameter discrete probability distribution with its neutrosophic extension: mathematical properties and applications. Int J Data Sci Analyt. 2023. https://doi.org/10.1007/s41060-023-00382-z.
- AlAita A, Talebi H. Exact neutrosophic analysis of missing value in augmented randomized complete block design. Comple Intell Syst. 2023. https://doi.org/10.1007/s40747-023-01182-5.
- Alhabib R, Ranna MM, Farah H, Salama A. Some neutrosophic probability distributions. Neutrosophic Sets Syst. 2018;22:30–8.
- Aliev RA, Alizadeh AV, Huseynov OH, Jabbarova K. Z-number-based linear programming. Int J Intell Syst. 2015;30(5):563–89.
- Alomair AM, Shahzad U. Neutrosophic mean estimation of sensitive and non-sensitive variables with robust hartleyross-type estimators. Axioms. 2023;12(6):578.
- 6. Aslam M. Data analysis for sequential contingencies under uncertainty. J Big Data. 2023;10(1):24.
- Bacanin N, Zivkovic M, Al-Turjman F, Venkatachalam K, Trojovský P, Strumberger I, Bezdan T. Hybridized sine cosine algorithm with convolutional neural networks dropout regularization application. Sci Rep. 2022;12(1):1–20.
- 8. Box G. ME Muller in Annals of Math. Stat. 1958;29:610-1.
- Chen J, Ye J, Du S. Scale effect and anisotropy analyzed for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics. Symmetry. 2017;9(10):208.
- Chen J, Ye J, Du S, Yong R. Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers. Symmetry. 2017;9(7):123.
- Chesneau C, Bakouch HS, Hussain T. A new class of probability distributions via cosine and sine functions with applications. Commun Stat-Simulation Comput. 2019;48(8):2287–300.
- Devroye L. A simple algorithm for generating random variates with a log-concave density. Computing. 1984;33(3):247–57.
- Jouhari H, Lei D, Al-qaness MAA, Abd Elaziz M, Ewees AA, Farouk O. Sine-cosine algorithm to enhance simulated annealing for unrelated parallel machine scheduling with setup times. Mathematics. 2019;7(11):1120.
- Kang B, Deng Y, Hewage K, Sadiq R. Generating Z-number based on OWA weights using maximum entropy. Int J Intell Syst. 2018;33(8):1745–55.
- Khan Z, Al-Bossly A, Almazah M, Alduais FS. On statistical development of neutrosophic gamma distribution with applications to complex data analysis. Complexity. 2021. https://doi.org/10.1155/2021/3701236.
- 16. Mirjalili S, Lewis A. The whale optimization algorithm. Adv Eng Softw. 2016;95:51–67.
- 17. Mohanta KK, Sharanappa DS, Mishra VN. Neutrosophic data envelopment analysis based on the possibilistic mean approach. Operat Res Decis. 2023;33(2). https://doi.org/10.37190/ord230205.
- 18. Mohazzabi P, Connolly MJ. An algorithm for generating random numbers with normal distribution. J Appl Mathemat Phys. 2019;7(11):2712–22.
- Muniappan A, Tirth V, Almujibah H, Alshahri AH, Koppula N. Deep convolutional neural network with sine cosine algorithm based wastewater treatment systems. Environ Res. 2023;219: 114910.

- 20. Polymenis A. A neutrosophic Student'st-type of statistic for AR (1) random processes. J Fuzzy Extension Appl. 2021;2(4):388–93.
- 21. Qiu D, Xing Y, Dong R. On ranking of continuous Z-numbers with generalized centroids and optimization problems based on Z-numbers. Int J Intell Syst. 2018;33(1):3–14.
- 22. Reju VG, Koh SN, Soon Y. Convolution using discrete sine and cosine transforms. IEEE Signal Process Lett. 2007;14(7):445–8.
- Sherwani RAK, Aslam M, Raza MA, Farooq M, Abid M, Tahir M. Neutrosophic normal probability distribution—a spine of parametric neutrosophic statistical tests: properties and applications neutrosophic operational research. Berlin: Springer; 2021. p. 153–69.
- 24. Smarandache F. Introduction to neutrosophic statistics, sitech and education publisher. Craiova: Romania-Educational Publisher; 2014. p. 123.
- 25. Smarandache F. Introduction to neutrosophic statistics: Infinite Study. Columbus: Romania-Educational Publisher; 2014.
- 26. Thomopoulos NT. Essentials of Monte Carlo simulation: statistical methods for building simulation models. Newyork: Springer; 2014.
- Wang P, Chen W, Lin S, Liu L, Sun Z, Zhang F. Consensus algorithm based on verifiable quantum random numbers. Int J Intell Syst. 2022. https://doi.org/10.1002/int.22865.

#### **Publisher's Note**

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

# Submit your manuscript to a SpringerOpen<sup>®</sup> journal and benefit from:

- ► Convenient online submission
- ► Rigorous peer review
- ► Open access: articles freely available online
- ► High visibility within the field
- Retaining the copyright to your article

Submit your next manuscript at > springeropen.com